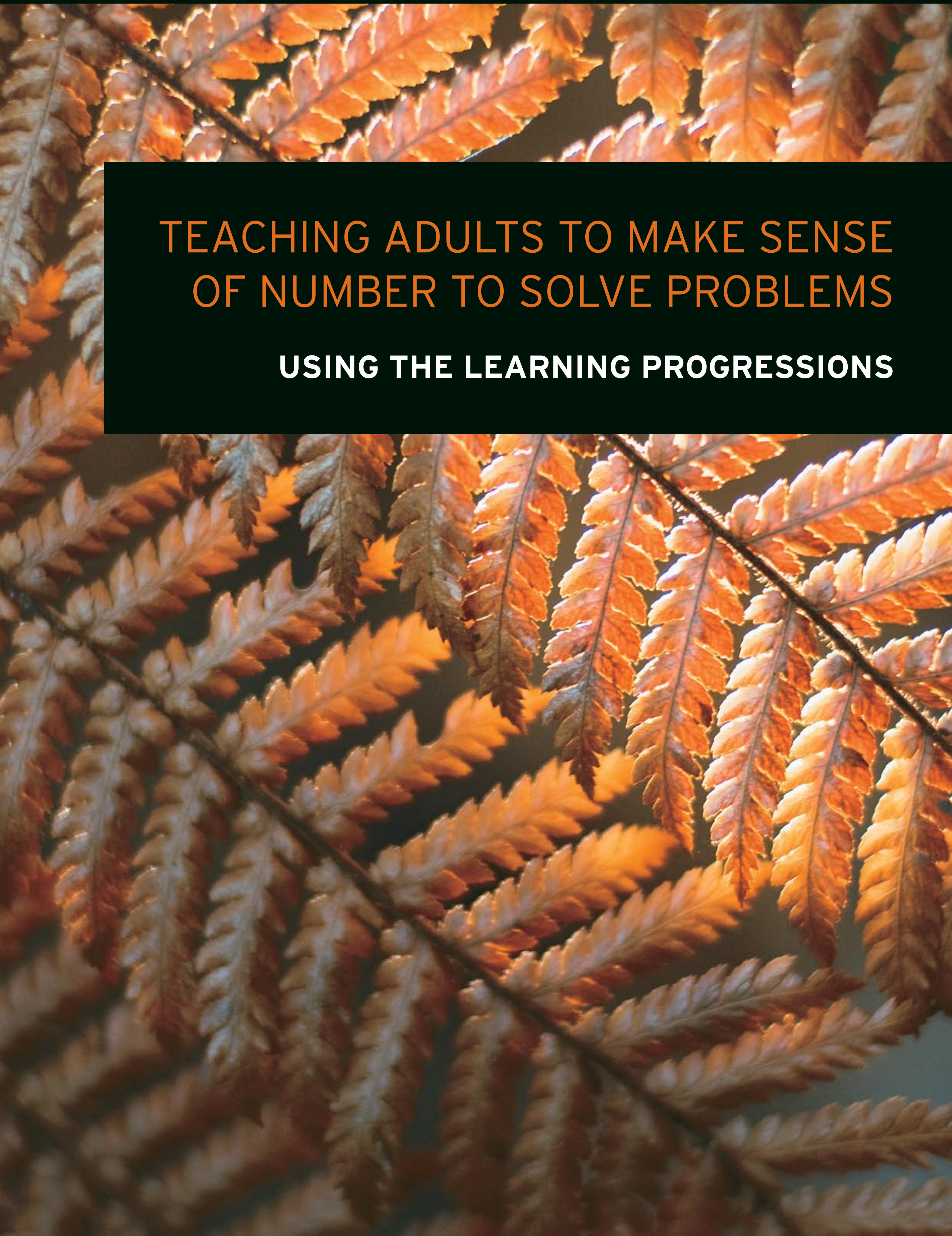
Tertiary Education Commission
Te Amorangi Mātauranga Matua

TEACHING ADULTS TO MAKE SENSE OF NUMBER TO SOLVE PROBLEMS

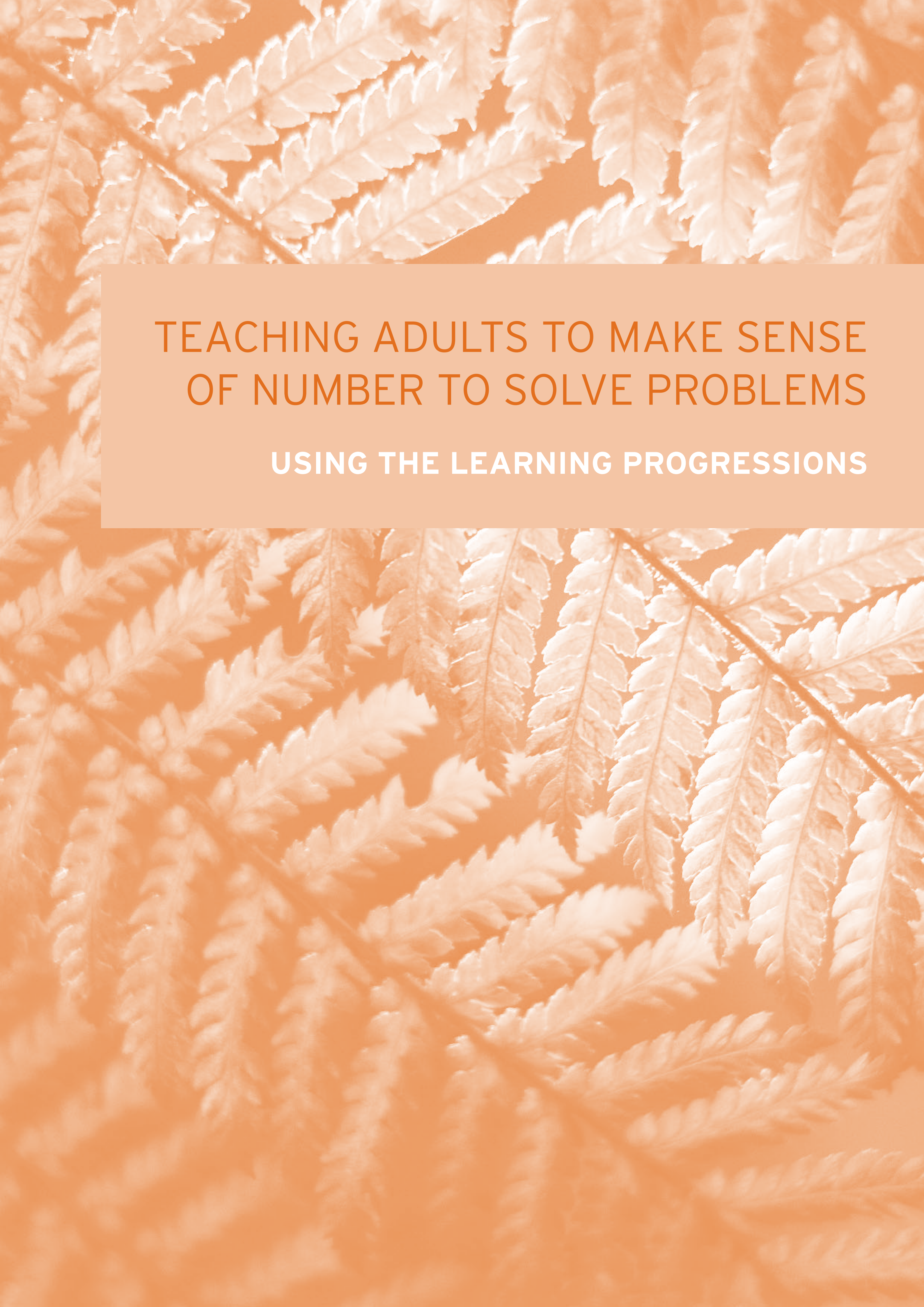
USING THE LEARNING PROGRESSIONS



Mā te mōhio ka ora:
mā te ora ka mōhio

Through learning there is life:
through life there is learning!

The Tertiary Education Commission would like to thank the many people who have contributed their time and expertise to this document, in preparation, design and development, consultation and review.



TEACHING ADULTS TO MAKE SENSE OF NUMBER TO SOLVE PROBLEMS

USING THE LEARNING PROGRESSIONS

Contents

Introduction

Teaching Adults to Make Sense of Number to Solve Problems: Using the Learning Progressions is part of a set of resources developed to support the teaching of literacy and numeracy for adult learners. The end goal is to enable tutors to meet the learning needs of their adult learners so those learners can engage effectively with the texts, tasks and practices they encounter in their training and learning. The suggestions in each booklet are aligned with the following Tertiary Education Commission (TEC) publications:

- *Learning Progressions for Adult Literacy and Numeracy: Background Information*
- *Learning Progressions for Adult Literacy*
- *Learning Progressions for Adult Numeracy.*

These can be located on the TEC website at www.tec.govt.nz.

These resources are based on research into effective adult literacy and numeracy, as described in *Lighting the Way*.[1] They also draw on school-sector work in literacy and numeracy, including Numeracy Project publications and the teachers' books *Effective Literacy Practice in Years 5 to 8* and *Effective Literacy Strategies in Years 9 to 13*.[2]

Readers are referred to the learning progressions publications (as listed above) to find more detailed discussions of adult learners, ESOL learners and the theoretical basis for each of the progressions. The progressions books also contain glossaries and reference lists.

This set of resources has been developed to support the learning progressions. The suggestions are initial ideas only: they are aimed at helping tutors apply the learning progressions to existing course and learning materials. It is expected that tutors will use, adapt and extend these ideas to meet the needs of learners and their own teaching situations. There are many other resources available for tutors to use, and comparisons with the learning progressions will help you determine where other resources may fit in your programmes, and how well they might contribute to learner progress.

1 Ministry of Education (2005). *Lighting the Way*. Wellington: Ministry of Education.

2 Ministry of Education (2006). *Effective Literacy Practice in Years 5 to 8*. Wellington: Learning Media Limited.
Ministry of Education (2004). *Effective Literacy Strategies in Years 9 to 13*. Wellington: Learning Media Limited.

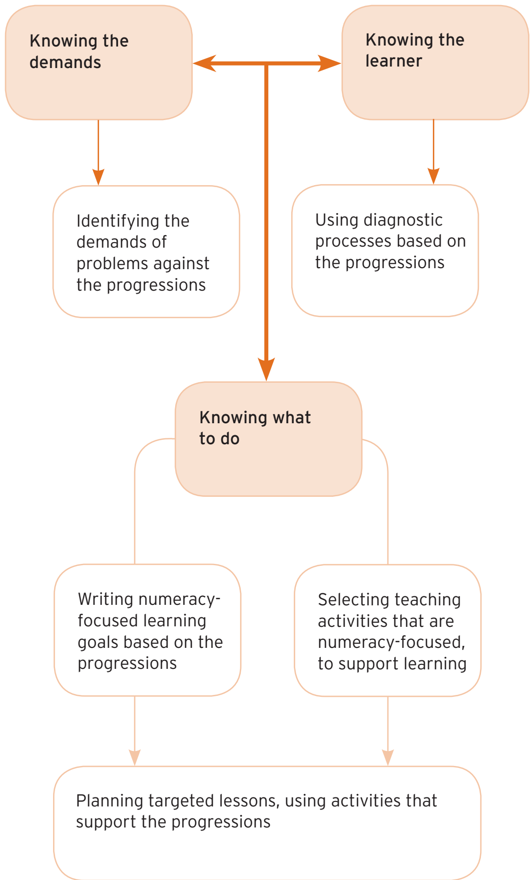
How to use this resource

There are three main sections in this resource:

- Knowing the demands (of the problems that learners want or need to manage).
- Knowing the learner (what they can do already, in order to determine the next learning steps).
- Knowing what to do (to help learners move on to the next steps).

These sections fit a process that can be illustrated as a flow chart.

Teaching adults to make sense of number to solve problems: using the learning progressions



It is not essential to follow this order – in some circumstances, it will make sense to start by getting to know the learners before finding out what it is that they want to be able to do.

The following guide to working with this resource should be used alongside the information in *Learning Progressions for Adult Numeracy.*

Knowing the demands

First, identify the numeracy demands of the number problems the learners need to be able to solve and map these against the learning progressions. There is a process that can be used to map problems as well as an applied example in this section, and a further three examples are included as Appendix A.

Knowing the learner

Use the tools in this section and the learning progressions to identify the learners' skills. Parts of this tool can be used with groups; other parts need to be carried out on an individual basis.

Knowing what to do

Use the learning progressions to set achievable goals for and with the learners. The decisions about what to teach should be based on the identified demands of the problems and where the learners 'sit' on the learning progressions. Identify specific activities and materials to use (based on your course and context), then apply them in your teaching. Finally, review and reflect on the outcomes for the learners, with the learners.

In this resource, mapping the problems the learners will encounter is the first step in planning for instruction. The next step is finding out where the learners 'sit' on the progressions. Where there is a gap between what the learners can do and what a task demands, you and your learners can refer to the learning progressions to make decisions about what to teach and learn next.

Strands and progressions

The learning progressions are organised within seven strands that cover the key components of listening, speaking, reading, writing and numeracy. Each progression shows a series of steps that reflects the typical sequence of skill development for oral language, written language and numeracy. The steps described are not tasks to be mastered in a set order. They do, however, offer information and a structure that can be used to develop curricula and learning and assessment tools. This current resource provides examples of how the progressions can be used. You are encouraged to design your own materials for teaching and learning to meet the needs of the adults with whom you work.

It is important to keep in mind that although the progressions are described in separate strands, in practice, we use literacy, language and numeracy skills and knowledge in ways that are typically interconnected. For example, a person may **listen** to a report about rising interest rates, **speak** to their partner about their mortgage, **read** the information from several banks (using their knowledge of **numbers** to interpret and compare rates), then **write** questions to ask a bank about the options for managing a mortgage. Even filling in a form requires both reading and writing skills, and may also involve a discussion to clarify terms or requirements. Learners will better understand how their existing knowledge can support new learning when these connections are made clear.

Knowing the demands

Adult learners need to learn to interpret and solve particular kinds of number problems for their individual purposes. As their tutor, you have to be able to analyse the problems the learners need to solve and identify the *demands* and supports that such problems present to the learners. This section provides a guide to mapping (analysing) the kinds of problems that adult learners need to be able to solve in relation to the learning progressions.

Mapping problems against the progressions

The learning progressions for numeracy reflect both number knowledge and strategies for solving number problems. The examples offered in this section and in Appendix A show how the demands of number problems can be mapped against these progressions. These examples show the approximate step (or difficulty level) that a learner would need to be working at in order to meet the demands of a particular task or problem.

To determine the challenges of the actual number problems that learners are expected to solve, you need to compare typical examples of these problems with the numeracy progressions and make decisions about each problem in relation to each progression. By comparing this information with what you know about the learners' knowledge and strategies skills, you will be able to determine the priorities for teaching and learning.

The examples provided are models to help you work out how to analyse the problems the learners will need to solve. Not every problem needs to be analysed in such detail, but it would be worthwhile analysing problems that are fundamental to a course.

Mapping process

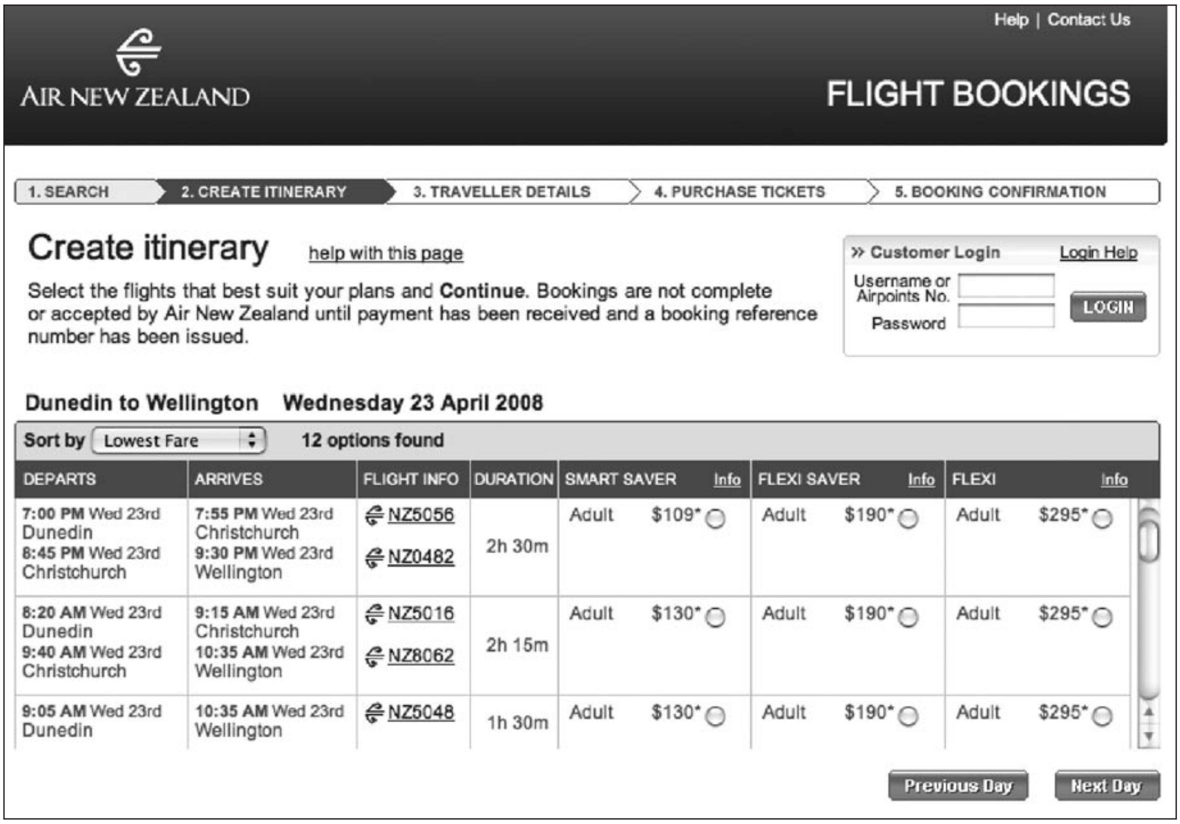
General process for all numeracy strands:

- Identify the strand or strands involved.
- Identify the progression or progressions involved.
- Identify the appropriate step in each applicable progression.

If the problem relates to the *Make Sense of Number* strand, ask yourself:

1. Does the problem involve working out a calculation and/or knowing facts about numbers?
2. If it involves a calculation, which of the three strategies progressions are applicable? If it involves knowing facts, which of the three knowledge progressions are applicable?
3. Which step on the progression is the 'best' fit for the size of numbers in the problem?

Analysing problems in this way will help you to decide the nature and amount of scaffolding (guided support) you will need to provide for the learners. For example, if you know that a learner does not have the knowledge of place value required by the problem, then this is what you need to teach to enable the learner to solve the problem.



Problem: Which flight is cheapest?

1. Does the problem involve working out a calculation or knowing facts about numbers?

   It involves knowing which number is largest, so it is one of the number knowledge progressions.

2. Which of the three number knowledge progressions?

   Number sequence.

3. What step is the best fit for the 'size' or 'type' of number that is involved?

   The problem involves whole numbers less than 1,000, 3rd step.

PROBLEM	SOLUTION	PROGRESSION(S) AND STEP(S)
Which fare is the cheapest?	$109 (I know this is less than $190 and $295.)	Number sequence, 3rd
How much more is a flexi than a flexi saver fare?	$295 - $190 = $105 i) 190 + 100 +5	Additive, 4th Number facts, 4th
What would be the total cost for three adults on a flexi fare?	$295 x 3 = $885 i) 300 x 3 = 900 900 - 15 = 885	Multiplicative, 4th Number facts, 4th
Fares include GST. What is the GST component on a flexi fare?	$295 ÷ 9 = $32.78 i. Using a calculator or algorithm, we can see that this answer is reasonable since: 30 x 9 = $270 or $300 ÷ 10 = $30 ii. To understand why you need to divide by 9 to find a GST amount. ($^{295}/_{112.5}$ x 12.5)	Multiplicative, 4th Number facts, 4th Proportional, 6th
If the 9.05am flight departs late at 9.45am, when will it arrive (assuming it is a direct flight)?	40 mins late. 40 minutes after 10.35am is 11.15am	Measurement, 4th

See Appendix A for more examples of mapped number problems.

Identifying complex demands

Many everyday work, social, personal and community activities present a wide variety of demands that cross over the (artificial) boundaries between 'numeracy', 'literacy' and other realms of knowledge and expertise. In order to make teaching and learning manageable, it may be necessary to break down a task (for example, conducting a stocktake and ordering from a catalogue) into component parts and then identify the task demands in relation to the learning progressions. You can go on to decide whether to address the demands one at a time or to select parts that can be addressed together, depending on the most immediate needs of the learner.

To use the stocktaking example noted above, the demands of reading a catalogue to find and record items could include:

- searching for specific information (Read with Understanding strand; *Language and Text Features* progression, 3rd step; *Comprehension* progression, 3rd step)
- knowing the sequence of numbers (Make Sense of Number to Solve Problems strand; *Number Sequence* progression, 3rd step)
- transferring information from the catalogue to an order form (Write to Communicate strand; *Planning and Composing* progression, 2nd step)
- finding the total price where multiples of an item are required (Make Sense of Number to Solve Problems strand; *Multiplicative Strategies* progression, 2nd or 3rd step).

There will no doubt be other factors involved in the task, but by identifying the demands in relation to the progressions, you can combine this information with information about the learning needs of individual learners (see Knowing the learner) to make decisions about what to teach first.

Knowing the learner

Teachers of adult learners need to be able to diagnose their learners' number strengths and needs in order to help those learners progress in their problem solving. The diagnostic assessment tool described below is based on the progressions in the *Make Sense of Number to Solve Problems* strand.

A diagnostic assessment tool

This diagnostic tool provides a procedure that has been designed to give you quality information about the number strategies and number knowledge of an adult foundation learner and to show where their knowledge and strategies fit within the learning progressions. There are some questions that you can use to help identify the learners' current attitudes towards learning mathematics, followed by tables that outline guided interview questions for strategies and for knowledge related to the steps within each progression.

Learner attitude questions

Ask these questions before starting the strategy assessment and record the learner's responses by circling the number that best matches their response to a question.

1. How much do you like maths?

HATE IT		OKAY		LOVE IT
1 ☐	2 ☐	3 ☐	4 ☐	5 ☐

2. How good do you think you are at maths?

AWFUL		OKAY		GREAT
1 ☐	2 ☐	3 ☐	4 ☐	5 ☐

3. How confident do you feel in doing maths during your daily life?

NOT AT ALL		OKAY		REALLY CONFIDENT
1 ☐	2 ☐	3 ☐	4 ☐	5 ☐

See Appendix C for a copy of the learner attitude questions.

General guidelines for using the diagnostic questions

- The questions in the tables that follow are given as a guide to help you work out what your learners know or don't know in relation to each of the *Make Sense of Number to Solve Problems* progressions.
- The strategy assessment needs to be conducted as an individual interview with each learner. The knowledge assessment can be conducted with a group or with individual learners.
- Use a 'best fit' approach for deciding where to place a learner on the progression.
- We suggest that you start with a diagnostic question from step 4 and then move up to steps 5 and 6 or back to earlier steps to identify the 'best fit' for the learner.
- Be prepared to create additional (similar) diagnostic questions if you are unsure where to place a learner or if the learner is already familiar with the example questions.
- If the learner is finding the questions too difficult, it is not necessary to ask all questions.

Additive Strategies progression

	DESCRIPTION	POSSIBLE DIAGNOSTIC QUESTIONS	COMMENTS TO TUTOR
	Solves add. and sub. problems by counting all of the objects	What is 8 + 5?	Notice if the learner uses fingers or objects to count 8 and 5 and starts from 1 to count up to 13.
	Solves add. and sub. problems by counting on or back using ones or tens	You have $27 and are given $9 more. How much do you have now? You have $42 and spend $8. How much do you have left?	Notice if the learner counts on in ones to solve this problem. 27 ... 28, 29, 30, 31, 32, 22, 34, 35, 36. If they count in ones ask if they can do it another way.
	Mentally solves two-digit and one-digit add. and sub. problems		Check if the learner uses a partitioning strategy to solve the problem: **27 + 9** 27 + 10 - 1 = 36 27 + 3+ 6 = 36 **42 - 8** 42 - 10 + 2 42 - 2 - 6 Knows 12 - 8 = 4 so 42 - 8 is 34
	Solves multi-digit add. and sub. problems with reasonableness	You have $610 and spend $98 of it. How much do you have left? You have $47 and are given $25 more. How much do you have now?	Check if the learner uses a partitioning strategy: **610 - 98** 610 - 100 + 2 **47 + 25** 47 + 3 + 22 47 + 20 + 5 40 + 20 + 7 + 5 50 + 25 - 3 Explains reasonableness if a calculator is used. 512 is reasonable as 98 is close to 100 and 610 - 100 = 510 72 is reasonable as 47 + 25 is close to 50 + 25 which is 75.
	Solves add. and sub. problems that involve decimals and integers with reasonableness	James ran 100 metres in 14.52 seconds. Ben took 0.9 seconds longer to run the same distance. How long did it take Ben to run 100 metres? Gordy ran 400 metres in 66.72 seconds. James was 8.34 seconds faster. How long did James take to run the same distance?	Check if the learner uses a partitioning strategy: **14.52 + 0.9** 14.52 + 1 - 0.1 = 15.42 or 0.52 + 0.9 = 1.42 14 + 1.42 = 15.42 Explains reasonableness if calculator is used. 58.38 is reasonable as 67 - 8 = 59, which is close to 58.38.
	Solves add. and sub. problems that involve fractions, with reasonableness	Tania and Harry buy two pizzas. Harry eats $^{3}/_{4}$ of a pizza and Tania eats $^{7}/_{8}$ of a pizza. How much pizza is left over altogether? Without calculating, which of the following four equations below gives the largest answer. Explain why. a) $^{1}/_{10} + ^{2}/_{3}$ b) $^{4}/_{5} + ^{3}/_{4}$ c) $^{9}/_{10} - ^{1}/_{8}$ d) $^{2}/_{5} + ^{2}/_{7}$	Check if the learner uses a partitioning strategy: **$^{3}/_{4} + ^{7}/_{8}$** $^{6}/_{8} + ^{7}/_{8} = ^{13}/_{8}$ so $^{3}/_{8}$ of a pizza was left. $^{1}/_{4} + ^{1}/_{8} = ^{3}/_{8}$ and 2/8 + 1/8 = 3/8 Check if the learner explains that $^{4}/_{5} + ^{3}/_{4}$ is greater than 1. All the others are less than 1.

Multiplicative Strategies progression

	DESCRIPTION	POSSIBLE DIAGNOSTIC QUESTIONS	COMMENTS TO TUTOR
	Solves mult. and div. problems by counting all of the objects	Robert has six $5 notes in his wallet. How much money does he have in total?	Notice if the learner uses fingers or objects to count from 1 (1, 2, 3, 4, 5, 6, ... 7, 8, 9, 10, 11, 12, 13, 14, 15, etc to 30)
	Solves mult. and div. problems by skip-counting		Notice if the learner skip-counts in fives to solve this problem. 5, 10, 15, 20, 25, 30
	Mentally solves single-digit mult. and div. problems by deriving from known facts	Simone knows that 8 x 8 = 64. How could she use this fact to work out what 72 divided by 8 equals?	Check if the learner knows how to derive unknown facts from known facts. 72 is 8 more than 64 so 72 is 9 x 8 so 72 divided by 8 = 9.
	Mentally solves mult. and div. problems with single-digit multipliers or divisors, with reasonableness	A stall has 6 trays of eggs for sale. There are 24 eggs in each tray. How many eggs are there in total?	Check if the learner uses a partitioning strategy: **6 x 24** (6 x 20) + (6 x 4) (6 x 2) x 12 (25 x 6) - 6
	Solves mult. and div. problems with multi-digit whole numbers, with reasonableness	A netball uniform costs $55. How much will 21 new uniforms cost the club? What is 870 x 102? Why is that answer reasonable?	Check if the learner uses a partitioning strategy: 20 x 55 = 1,100 1 x 55 = 55 1,100 + 55 = $1,155 Check if the learner can reason that 870 x 102 is going to be close to but larger than 87,000.
	Solves mult. and div. problems that involve decimals, fractions and percentages, with reasonableness	Sez used a calculator and found that 49 ÷ 0.098 = 500. Explain why the answer is reasonable. The full cost of a bag of potatoes is $4.50. This week there is 10% off the cost. What is the cost of potatoes this week?	Check if the learner can reason that: 49 divided by a number close to a tenth is going to give a number about 10 times larger than 49 (close to 490). 10% of $4.50 is 45 cents and 45 from $4.50 is $4.05. It is reasonable if the learner states $4 or $4.10 as this will be the amount charged if cash is used.

Proportional Reasoning Strategies progression

	DESCRIPTION	POSSIBLE DIAGNOSTIC QUESTIONS	COMMENTS TO TUTOR
	Finds a fraction of a set by using equal sharing	There are 12 cans of drink. You are given $^1/_3$ of them. How many cans are you given?	Notice if the learner uses objects to share 12 into 3 equal groups or if they know that strategy. Notice if the learner uses knowledge of basic facts (3 x 4 = 12 or 12 ÷ 3 = 4) to solve this problem.
	Uses mult. and div. strategies to find unit fractions of whole numbers	What is $^1/_4$ of 64?	Check if the learner uses a partitioning strategy: **$^1/_4$ of 64** 4 x 10 = 40, 4 x 6 = 24 $^1/_2$ of 64 = 32, $^1/_2$ of 32 = 16 $^{60}/_4$ = 15 so $^{64}/_4$ = 16.
	Uses mult. and div. strategies: equiv. fractions, simple conversions between fractions, decimals, percentages	Jeans cost $70. If they are reduced by 15%, how much do you save? What is $^2/_3$ of 330?	Check if the learner uses a partitioning strategy: **15% of 70** 10% is $7, 5% is $3.50, so there is a $10.50 saving. **$^2/_3$ of 330** $^1/_3$ of 330 is 110, $^2/_3$ is 220.
	Uses mult. and div. strategies: problems involving proportions, ratios and rates	An item that used to sell for $50 now costs $76. What percentage increase is this? A car travels 150 kilometres on 10 litres of petrol. How many litres of petrol does it take to travel 60 kilometres?	Check if the learner uses a partitioning strategy: **$50 to $76 =** $^{26}/_{50}$ increase, which is equivalent to $^{52}/_{100}$; so a 52% increase. The car travels 15 kilometres on each litre so it takes 4 litres to travel 60 kilometres. or 10:150 ?:60 10:150 is 2:30 or 4:60

Number Sequence progression

	DESCRIPTION	POSSIBLE DIAGNOSTIC QUESTIONS	COMMENTS TO TUTOR
	Sequence to 20	What number is one more than 9? What number is one less than 16?	Check that the learner knows the sequence (forward and back) of numbers to 20.
	Sequence to 100 Skip-count 2, 5, 10	What number is one more than 89? What number is one less than 50? Continue this pattern 5, 10, 15 …	Check that the learner knows the sequence (forward and back) of numbers to 100. Check if the learner can skip-count in fives.
	Sequence to 1,000 Order fractions	What number is ten more than 499? What number is ten less than 843? Put these fractions in order from the smallest to the largest. 2/6, 5/6, 1/6	Check that the learner knows the sequence (forward and back) of numbers to 1,000. Check that the learner can give a number ten less than a number given. Check that the learner can order fractions of the same denominator.
	Sequence to 1,000,000 Order unit fractions	What number is one more than 989,999? What number is one less than 603,000? Put these fractions in order from the smallest to the largest. 1/10, 1/2, 1/5	Check that the learner knows the sequence of numbers to 1,000,000. Check that the learner can order unit fractions.
	Sequence of decimals, percentages and fractions	Put these in order from smallest to largest. 20%, 0.259, 1/4, 0.21	Check if the learner can sequence fractions, decimals and percentages.

Place Value progression

	DESCRIPTION	POSSIBLE DIAGNOSTIC QUESTIONS	COMMENTS TO TUTOR
	Place value to 100	A CD player costs $80. How many $10 notes do you need to pay for it?	Check that the learner knows how many tens in 80.
	Place value to 1,000	A TV costs $470. How many $10 notes do you need to pay for it?	Check that the learner knows how many tens in 470.
	Place value to 1,000,000	A car costs $45,400. How many $100 notes do you need to pay for it?	Check that the learner knows how many hundreds in 45,400.
	Tenths, hundredths, thousandths Convert dec. to % Powers of 10	Write a number that lies between 7.59 and 7.6. What is 135% as a decimal? $10^3 = ?$	Check that the learner knows how to use thousandths. Check that the learner can convert % to decimals. Check if the learner knows powers of 10.

Number Facts progression

	DESCRIPTION	POSSIBLE DIAGNOSTIC QUESTIONS	COMMENTS TO TUTOR
	Sums of 5, 10 decade facts	2 + 3 = 10 - 6 = 50 + 7 =	
	Add. and sub. Facts to 10 + 10	9 + 9 = 13 - 7 =	
	Mult. and div. Facts to 10 x 10	6 x 4 = 8 x 7 = 27 ÷ 3 =	
	Mult. and div. with 10, 100, 1,000	200 x 80 = 5 million ÷ 10 =	
	Common factors Exponents	2^3 = ? What is $^3/_4$ as a %? What factors do 18 and 30 have in common?	

Knowing what to do

General principles for guided teaching and learning activities

Each activity is aligned to a step on one of the *Make Sense of Number* progressions and is intended to strengthen the learners' understandings of the concepts and skills associated with that step. The activities assume that the learners understand the concepts and skills from earlier steps in the progression, so it is important you check that the learners have these understandings. Each teaching and learning activity includes the following components:

- A statement summarising the mathematical understandings that you will support the learners to develop in the activity.
- A list describing the teaching points covered in the activity.
- A list of resources or materials used in the activity. (These resources are readily available within adult learning settings and do not require the purchase of specialised materials.)
- A guided teaching and learning sequence that details the steps for you to use to develop the learners' knowledge of the skills and concepts addressed. The sequence includes questions for you to pose to the learners. The questions are intended to help the learners clarify their understandings as they explain their thinking to others. The questions are also designed to help you scaffold the learners' understanding of the concept or skill being explored.
- A follow-up activity for the learners to complete independently.

Summary of guided teaching and learning activities

There are currently 30 activities for *Make Sense of Number to Solve Problems*; some of them serve more than one progression. The progressions and steps that each activity relates to are indicated in the chart and noted at the start of each activity. Note that further activities may be added from time to time.

Activities for teaching and learning

Number Strategies progressions

	ADDITIVE	MULTIPLICATIVE	PROPORTIONAL REASONING
	Counting on and back, page 19	Skip-counting, page 32	
	Addition and subtraction strategies I, page 22	Understanding multiplication, page 35 Deriving multiplication and division facts, page 37	
	Addition and subtraction strategies II, page 25	Multiplication strategies, page 39 Division strategies, page 41	Ratios I, page 53 Fractions of numbers I, page 55
	Adding decimals, page 28 Subtracting decimals, page 30	Multiplying options, page 43 Dividing options, page 46	Ratios II, page 58 Fractions of numbers II, page 61
		Multiplying with decimals, page 49 Dividing with decimals, page 51	Rates and proportions, page 64

Number Knowledge progressions

	NUMBER SEQUENCE	PLACE VALUE	NUMBER FACTS
	Numbers to 100, page 67	Introducing place value, page 73	Addition and subtraction facts, page 80
	Understanding fractions I, page 69	Whole number place value, page 76	Multiplication and division facts, page 82 See also: Understanding multiplication, page 35 Deriving multiplication and division facts, page 37
	Understanding fractions II, page 70 Understanding fractions III, page 71		Estimating facts, page 85
	See also: Decimal number place value, page 78	Decimal number place value, page 78 See also: Connecting percentages, decimals and fractions, page 87	Connecting percentages, decimals and fractions, page 87

Counting on and back

Additive Strategies progression, 2nd step

The purpose of the activity

In this activity, the learners develop their understanding of addition and subtraction by counting on in ones to solve problems. They learn that you do not have to start counting from one to solve problems.

The teaching points

- Counting tells how many objects there are in a set. The last word in the counting sequence tells how many objects are in the set.
- When counting on, start from the largest number. For example, count on from 18 rather than 4 to solve 4 + 18 = ?
- Addition and subtraction are closely related. Addition names the whole in terms of its parts. Subtraction names the missing part when the whole and one of the parts is known.
- Addition problems have a variety of types or structures. It is important that the learners solve problems of all types to strengthen their understanding of addition.
    - Result unknown. In this type of problem, two numbers are given, and you have to find the result (for example, 4 + 6 = ?).
    - Change unknown. In this type of problem, an initial and a final number are given, and you have to find the number in between (for example, 5 + ? = 10).
    - Start unknown. In this type of problem, you know what has happened to an unknown number to give a particular answer. You have to find the starting number (for example, ? + 5 = 7).
- Subtraction problems have a variety of types or structures. It is important that the learners solve problems of all types to strengthen their understanding of subtraction.
    - Result unknown. In this type of problem, you know the largest or the starting amount and the amount that has been taken away. You need to find the resulting amount (for example, 20 – 5 = ?).
    - Change unknown. In this type of problem, the starting amount and the amount that is left are given. You need to find the amount that was taken away (for example, 20 – ? = 15).
    - Start unknown. In this type of problem, the amount taken away and the resulting amount are known. You have to find the starting amount (for example, ? – 5 = 15).
- Discuss with the learners the contexts or situations where they need to use addition and subtraction.

Resources

- Sets of digit cards (10 cards labelled from 0 to 9).

The guided teaching and learning sequence

1. Show the learners the set of digit cards. Ask a volunteer to select a card from the set. Record this number on the board (for example, 4).
2. Ask another volunteer to select two cards from the set of digit cards and form a two-digit number. Record this number on the board (for example, 45).
3. Tell the learners that you want them to join or add the two numbers together. Record 4 + 45 = on the board.
4. Ask the learners to first work out the answer and then think about how they would explain what to do to someone who was having difficulty working it out.

    "What would you say to someone who was having difficulty working out 4 + 45?"

continued...

5. Ask the learners to share their solutions, ensuring they:
   - start with the largest number (for example, 4 + 45 becomes 45 + 4)
   - use the counting sequence to count on from the largest number (for example, 46, 47, 48, 49).
6. Give pairs of learners a set of digit cards. Explain that they are to take turns selecting the numbers and solving the problems posed. The learner who selects the cards first places them in front of the other learner and asks that person to join or add the numbers together. The learner who is solving the problem must explain how they found the answer to their partner and record the problem and answer on a sheet of paper.

9		3	4

34 + 9 = 43
etc

7. Repeat the exercise a couple of times. As the learners work together, circulate, checking that the learners are beginning with the largest number and that they understand the sequence of numbers used.
8. Ask a volunteer to draw two digits from the set of digit cards and form the largest number possible with these two digits. Doing this provides you with another opportunity to assess the learners' understanding of numbers. Record the number on the board (for example, 62).
9. Tell the learners that this time they are going to subtract or take amounts away from 62. Ask a volunteer to select a card from the set of digit cards to indicate how much is to be taken away from 62 (for example, 6). As a class, count back 6 from 62 to find the answer to 62 – 6 = ?
10. Check the learners understand that they count: 61, 60, 59, 58, 57, 56. Record 62 – 6 = 56 on the board.
11. Give pairs of learners a set of digit cards. Explain they are to take turns selecting the numbers and solving the problems posed. The learner who selects the cards first places them in front of the other learner and asks that person to subtract the single-digit number from the double-digit number. The learner who is solving the problem must explain how they found the answer to their partner and record the problem and answer on a sheet of paper.

4	1		9

41 - 9 = 32
etc

12. Repeat the exercise a couple of times. As the learners work together, circulate, checking that the learners are correctly counting back to solve the problem.

The rest of this guided learning sequence repeats the above steps with different types of addition and subtraction problems. These could be taken as separate teaching sessions.

13. Write the following **addition start unknown** problem on the board:

    "Sandy is collecting donations for a charity. A person gives her $7. She now has $71. How much did she have to start with?"

14. Discuss with the learners how this problem can be recorded as ? + 7 = 71.

15. Ask the learners to think about how they would solve the problem. Ask for volunteers to share their solutions. For example "I counted back from 71 to get to 64". Ensure the learners understand this approach means that they have used subtraction (or counting back) to solve the addition problem.

16. Ask the learners to work in pairs to pose problems to one another where the starting amount is unknown, using numbers drawn from the set of digit cards (as in step 6 above). Alternatively, the learners could work as a group to solve the following problems:

    - ? + 6 = 78
    - ? + 9 = 35
    - ? + 2 = 40

17. Write the following **addition change unknown** problem on the board:

    "Sandy is collecting donations for a charity and has $79. A person makes a donation. She now has $85. How much did the person donate?"

18. Discuss with the learners how this problem can be recorded as 79 + ? = 85.

19. Ask the learners to think about how they would solve the problem. Ask for volunteers to share their solutions. For example "I counted on 6 from 79 to get to 85" or "I counted 6 back from 85 to get 79". Ask the learners to work in pairs to pose problems to one another where the starting amount is unknown, using numbers drawn from the set of digit cards (as in step 6 above). Alternatively, the learners could work as a group to solve the following problems:

    - 36 + ? = 45
    - 12 + ? = 19
    - 75 + ? = 82

Follow-up activity

Ask the learners to write addition and subtraction problems that involve two-digit and one-digit numbers on strips of paper for others to solve. On the reverse side of the strip of paper, have the learners write the solution and the counting on or back strategy they used to solve the problem.

35 + ? = 43

36, 37, 38, 39, 40, 41, 42, 43 35 + 8 = 43 Counting on

Addition and subtraction strategies I

Additive Strategies progression, 3rd step

The purpose of this activity

In this activity, the learners use strings of beads and/or number lines to develop addition and subtraction mental partitioning strategies for two-digit by one-digit problems, for example, 27 + 8, 36 – 7.

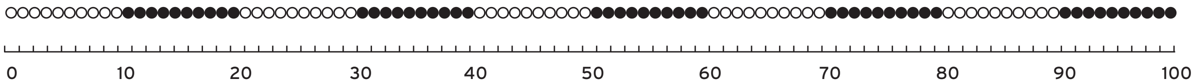
The teaching points

- Learners who 'count on' (for example, calculating 27 + 8 by counting 27, 28, 29, 30, etc, either using fingers or in their heads) or who can only use a calculator to make calculations will be disadvantaged in completing the numeracy demands of everyday life.
- It is not anticipated that you will teach a learner many different strategies for solving a single problem, but rather you will work with one strategy at a time, noting that different problems lend themselves to different strategies.
- Learners need to know addition and subtraction facts to 10 + 10 and the place value of digits in whole numbers to 100 before undertaking this activity.
- Discuss the strategies and how they can be used with the learners.
- Discuss with the learners the contexts or situations where they need to solve addition and subtraction problems mentally.

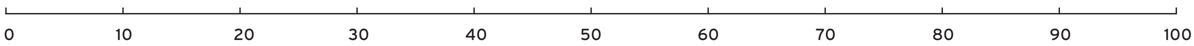
Note: This learning sequence can be used with single-digit addition and subtraction to help the learners with their basic 10 + 10 addition and subtraction facts.

Resources

- 100 beads, 50 of one colour and 50 of another colour, threaded on to a string in blocks of 10 for each colour or number lines marked in ones.



- Number lines marked in tens.

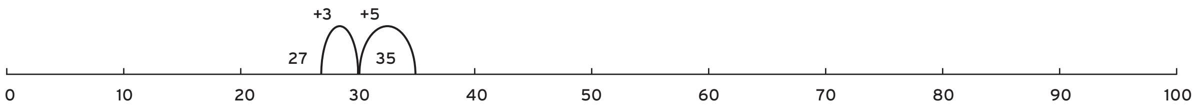


The guided teaching and learning sequence

1. Ask the learner(s) to use the string of beads or the number lines marked in ones to solve 27 + 8 in any way they can and explain what they have done. If the learner 'counts on', ask if they can see any other way to solve the problem.
    - If the learner(s) shows evidence of some partitioning strategy, ask them to demonstrate on the beads and continue to develop that strategy throughout the activity.
    - If the learner(s) shows no evidence of using partitioning strategies, encourage them to use the strategy of 'making tens' by:
        - asking the learner(s) how many beads are needed to get from 27 to 30 and marking + 3 on the number line
        - asking how many remain of the 8 and marking + 5 on the number line.
2. Repeat the process with a variety of different two-digit by one-digit addition problems.

continued...

3. Once the learner(s) is familiar with the process, ask them to do the activity on the number line marked in tens (0, 10, 20, 30, 40 ... 100), for example, starting at 27 on the line and marking + 3 to 30 and + 5 to 35.



4. Repeat the process with a variety of two-digit by one-digit addition problems.
5. Once the learners are familiar with the process, repeat each of the steps for two-digit by one-digit subtraction problems, for example 45 – 7.

Follow-up activity

In pairs, the learners make up and solve two-digit by one-digit addition and subtraction problems, sharing the strategies they use with their partner.

Addition and subtraction strategies II

Additive Strategies progression, 4th step

The purpose of this activity

In this activity, you will use number lines to encourage the learners to become familiar with a variety of addition and subtraction mental partitioning strategies for multi-digit problems, for example 26 + 27, 47 – 38.

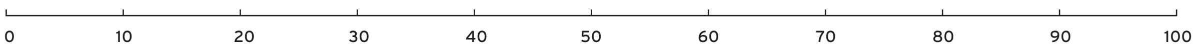
The learners need to be familiar with the concepts addressed in the "Addition and subtraction strategies I" activity before starting this activity.

The teaching points

- There are a variety of different partitioning strategies (see *Learning Progressions for Adult Numeracy* for examples.)
- Different problems lend themselves to different strategies, and competent learners have a range of strategies to choose from.
- Number lines marked in tens and then empty number lines are useful materials for developing mental strategies.
- Discuss with learners the situations where they need to solve addition and subtraction problems and the advantages of being able to do this mentally.

Resources

- Number lines marked up in tens.



- Empty number lines.

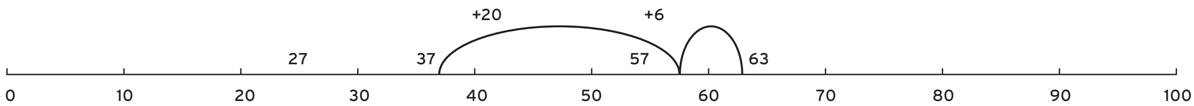
continued...

The guided teaching and learning sequence

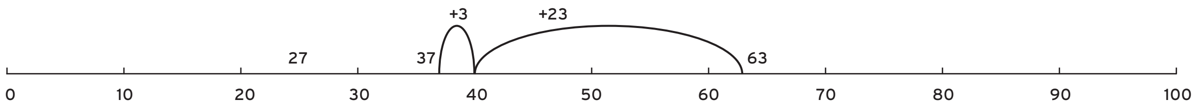
1. Ask the learners individually to use number lines marked with tens to solve 37 + 26 in any way they can and then explain what they have done to the whole group.
2. Listen for different strategies and record them on the board on number lines marked with tens.

   For example:

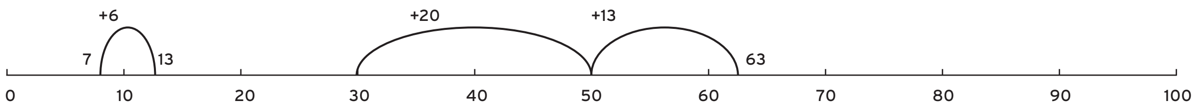
   - 37 + 20 = 57 and 6 more make 63.



   - 37 + 3 = 40, and 23 more makes 63.



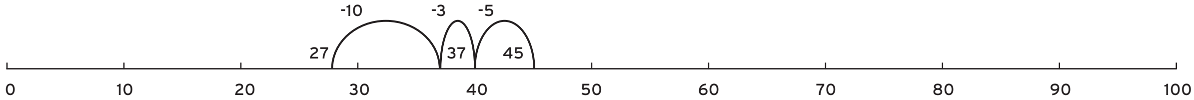
   - 30 + 20 makes 50 and 7 + 6 = 13 and 50 + 13 makes 63.



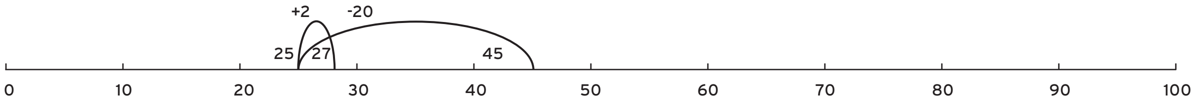
3. Ask the learners to work in groups, using marked number lines to solve similar problems, for example, 26 + 27, 14 + 39 and 62 + 35, and to demonstrate and share as many strategies as possible. Ask them to consider whether different strategies are easier for different problems.
4. Once the learners are demonstrating a variety of strategies for addition, repeat steps 1 to 3 above with subtraction.

   An example is 45 – 18 and possible strategies include:

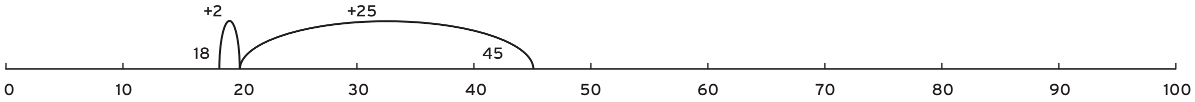
   - 45 – 5 = 40, subtract 3 more makes 37 and another 10 makes 27.



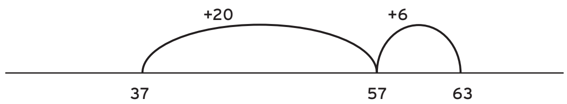
- 45 – 20 = 25 and add 2 makes 27.



- 18 + 2 = 20 and another 25 makes 45 : 27 is added in total.



5. When the learners are familiar with using number lines marked in tens, repeat the above steps with empty number lines.
   - For example, 37 + 26.



6. When the learners are familiar with using unmarked number lines, challenge them to solve problems mentally, giving answers and describing the strategies they used. Encourage any learners who are having difficulty to use an empty number line.

Follow-up activity

Give the learners cards with one addition or subtraction problem on each card. Working one on one or with the learners in pairs or in groups, ask the learners to solve the problem and explain the strategy they used.

Adding decimals

Additive Strategies progression, 5th step

The purpose of this activity

In this activity, the learners use strategies, traditional written methods and calculators to solve addition problems that contain decimal fractions. The aim of exploring the three calculating approaches is to encourage the learners to anticipate from the complexity and structure of a problem the approach that best suits the problem and them.

The learners need to be familiar with the concepts addressed in the "Addition and subtraction strategies I and II" activities before starting this activity.

The teaching points

- Addition and subtraction with decimals builds on the same concept used with whole numbers, that is, you add or subtract numbers of like position values. For example, hundredths are added to or subtracted from hundredths.
- Different problems lend themselves to different strategies, and competent learners have a range of strategies to choose from.
- There are three main calculating approaches: using strategies to calculate a problem mentally, using traditional written methods (algorithms) and using calculators. The complexity of the problem determines the most effective and efficient calculating approach.
- Irrespective of the calculating approach used, it is important that learners are able to judge the reasonableness of their answer in relation to the problem posed.
- The decimal point is a convention that indicates the units, place. The role of the decimal point is to indicate the units, or ones, place in a number, and it does that by sitting immediately to the right of that place. Consequently the decimal point also works to separate the units (on the left) from parts of the unit (on the right).
- Discuss with the learners relevant or authentic situations where the addition and subtraction of decimals occurs (for example, carpentry when measurements are given as parts of metres, time when measured to milliseconds).

Resources

- Empty number lines.
- Decimal number lines.
- Calculators.

The guided teaching and learning sequence

1. Write the following problem on the board: Sian ran 400 metres in 56.52 seconds. Jason took 1.3 seconds longer. How long did it take Jason to run 100 metres?
2. Discuss with the learners the three options they have for solving the problem:
   - using a calculator
   - using a written method
   - using a strategy.
3. Ask the learners to choose one of the options and solve the problem giving them 2 to 3 minutes to do this. Then ask them to share their solution and how they solved it with another learner. Remind them they also have to be able to explain why the answer they obtained is reasonable (or makes sense) in relation to the problem.
4. Ask the learners to indicate (with a show of hands) which of the three approaches they used. As this is a relatively simple decimal addition, the learners should be able to calculate it mentally, using a strategy.

5. Ask for a volunteer who used a calculator to solve the problem.

   "Why did you choose to use a calculator?"
   "Why is your answer reasonable?"

   Encourage the learners to notice that 57.82 is reasonable because it is about 1 second more than 56.52.

6. Ask for a volunteer who used a written method to solve the problem.

   "Why did you use that method?"
   "Show us what you did on the board."
   "How did you know that your answer was reasonable?"

   Check that the learners have appropriately lined up the places or positions in each number. If the learner has added a 0 to 1.3, ensure everyone understands its use as a 'place holder'. This also provides an opportunity to talk about the decimal point and its use as an 'indicator' of the ones, or units, place.

56.52 1.3 57.82	56.52 1.30 57.82

7. Ask for a volunteer who used a mental strategy to solve the problem.

   "Why did you decide to use a mental strategy?"
   "Explain what you did to work out the answer."
   "How did you know that your answer was reasonable?"

   56.52 + 1 + .3 = 57.52 + .3 = 57.82

   56 + 1 + .5 + .3 + .02 = 57 + .8 + .02 = 57.82

8. Pose another problem:
   Susan took 63.34 seconds to run 400 metres. Kris ran 1.99 seconds faster. How long did Kris take to run the 400 metres.

9. This time ask the learners to work in pairs to solve the problem, using the three different approaches suggested above. As the learners solve the problem, discuss with them their preferred approach.

10. Give the learners the following problems on a sheet of paper. Without actually solving the problems, ask the learners to look at each problem and write down which approach they think they would prefer to use to solve the problem.

Which approach will you use?	
a.	7.13 + 8.24
b.	3.22 + 3.47
c.	4.966 + 1.644
d.	15 + 1.212
e.	25.04 + 25.704

11. As a class, discuss which problems seem best suited to mental strategies and which they would prefer to solve with a calculator or written method.

Follow-up activity

Give the learners cards that have a decimal addition problem on each card. Have the learners work in pairs to select a computation approach, solve the problem and explain the reasonableness of their answer.

Subtracting decimals

Additive Strategies progression, 5th step

The purpose of the activity

In this activity, the learners use strategies, traditional written methods and calculators to solve subtraction problems that contain decimal fractions. The aim of exploring the three computation options is to encourage the learners to anticipate from the complexity and structure of a problem the approach that is best suited to that problem.

The learners should be familiar with the concepts addressed in the "Addition and subtraction strategies I and II" activities before starting this activity. This activity could follow or be completed in conjunction with the "Adding decimals" activity.

The teaching points

- Addition and subtraction with decimals builds on the same concept used with whole numbers, that is, you add or subtract numbers of like position values. For example, hundredths are added to or subtracted from hundredths.
- Different problems lend themselves to different strategies, and competent learners have a range of strategies to choose from.
- There are three main calculating approaches: using strategies to calculate a problem mentally, using traditional written methods (algorithms) and using calculators. The complexity of the problem determines the most effective and efficient calculating approach.
- Irrespective of the computation approach used, it is important that learners are able to judge the reasonableness of their answer in relation to the problem posed.
- The decimal point is a convention that indicates the units, place. The role of the decimal point is to indicate the units, or ones, place in a number, and it does that by sitting immediately to the right of that place. Consequently the decimal point also works to separate the units (on the left) from parts of the unit (on the right).
- Like any other digit, 0 indicates the number of items in the place (or column) in which it appears. The 0 can also be considered a place holder. For example, in 6.05, the 0 holds the tenths place so that the 5 appears in the hundredths place. The 0 is not needed as a place holder when it is not between a digit and the decimal point, for example 1.50 and 1.5 are the same.
- Discuss with learners relevant or authentic situations where the addition and subtraction of decimals occurs (for example, carpentry when measurements are given as parts of metres, time when measured to milliseconds).

Resources

- Empty number lines.
- Decimal number lines (masters available from nzmaths website www.nzmaths.co.nz/numeracy/materialmasters.aspx).
- Calculators.

The guided teaching and learning sequence

1. Write the following problem on the board: 31.37 – 29.27 =

   Discuss with the learners real-life situations for that calculation. Possibilities include calculations that involve money, measurements in metres or kilograms, time. For example, Kate swam the first 50 metres in 31.37 seconds and the second 50 metres in 29.27 seconds. How much faster was her second 50 metres?

2. Discuss with the learners the three options they have for solving the problem:
   - using a calculator
   - using a written method
   - using a strategy.

3. Ask the learners to choose the option that is best suited to them for solving this problem. After giving them time to solve the problem, ask them to share how they solved it with another learner. Remind them they also have to be able to explain why their answer is reasonable (or makes sense) in relation to the problem.
4. Ask the learners to indicate (with a show of hands) which of the three approaches they used. As this is a relatively simple decimal subtraction, the learners should be able to calculate it mentally, using a strategy.
5. Ask for a volunteer who used a calculator to solve the problem.

   "Why did you choose to use a calculator?"
   "Why is your answer reasonable?" (Encourage the learners to notice that 2.1 is reasonable as 31 is 2 more than 29.)
6. Ask for a volunteer who used a written method to solve the problem.

   "Why did you use that method?"
   "Show us what you did on the board."
   "How did you know that your answer was reasonable?"

   Check that the learner has appropriately lined up the places or positions in each number. This also provides an opportunity to talk about the decimal point and its use as an 'indicator' of the ones or units position.

$$\begin{array}{r} 31.37 \\ -29.27 \\ \hline 2.10 \end{array}$$

7. Ask for a volunteer who used a mental strategy to solve the problem.

   "Why did you decide to use a strategy?"
   "Explain what you did to work out the answer."
   "How did you know that your answer was reasonable?"

   - 29.27 + 2 + 0.1 = 31.37
     (solving as an addition problem)
8. Pose another problem:

   - 55.63 – 42.97 =

   This time, ask the learners to work in pairs to solve the problem, using the three different approaches. As the learners solve the problem, discuss with them their preferred approach.
9. Give the learners the following problems on a sheet of paper. Without actually solving the problems, ask the learners to look at each problem and write down which approach they think they would prefer to use to solve the problem.

Which approach will you use?	
a.	6.78 – 0.23
b.	15.99 – 5.5
c.	4.3 – 2.29
d.	8.675 – 4.289
e.	24 – 6.572

10. As a class, discuss which problems seem best suited to mental strategies and which they would prefer to solve with a calculator or written method.

Follow-up activity

Give the learners cards with a decimal subtraction problem on each card. Have the learners work in pairs to select a computation approach, solve the problem and explain the reasonableness of their answer.

Skip-counting

Multiplicative Strategies progression, 2nd step

The purpose of the activity

In this activity, the learners develop their understanding of multiplication by skip-counting in twos, threes and fives to solve simple problems. By doing this, they begin to learn the 2, 3 and 5 times tables.

The teaching points

- Multiplication involves counting groups of a like size to find out how many there are altogether.
- Multiplication and division are connected. Multiplication names the product of two factors. Division finds a missing factor when the product and the other factor are known.
- The usual convention is that 4 x 2 refers to four sets of 2 rather than 2 sets of 4, although there is no reason to be rigid about this. The important idea is that the learners can relate the factors in the equation to the problem context.
- Discuss the ways learners can use skip-counting in everyday situations.

Resources

- A 10 x 10 grid numbered from 1 to 100.
- Highlighters.

The guided teaching and learning sequence

1. Ask the learners to work out how many human legs there are in the classroom (for this example, assume there are 14 people in the room).

   "There are 14 of us in the room. How many legs are there altogether?"

2. Tell the learners to stand. Ask:

   "How can we count legs to solve the problem?"

3. If a learner suggests counting in ones, do that by asking each person in turn to count their legs and then sit down. For example, the first person counts "1, 2" (then sits down), the second person counts "3, 4" (then sits down) and so on to 28.
4. Hopefully someone will suggest counting in twos. Once more, ask the learners to stand and take turns 'skip-counting' in twos. For example, the first learner says 2 (then sits down), the second leaner says 4 (then sits down) and so on to 28.
5. Give each learner a 10 x 10 grid and ask them to use a highlighter to shade in the multiples of 2 starting from 2.
6. When they have finished, ask:

   "What do you notice about the numbers when you count (or skip-count) in twos?"

   - They all end in 0, 2, 4, 6, or 8 (that is, even numbers).
   - They form columns on the chart.

7. "How could you use this chart to work out 7 lots of 2?" (Use the chart to help keep track of the count of twos)
8. Ask the learners to work in pairs to solve the following problems by skip-counting in twos. Encourage them to use the chart they developed earlier if they have difficulty keeping track of the number of twos they have counted.

   - 3 lots of 2
   - 8 lots of 2
   - 20 lots of 2
   - 33 lots of 2.

The next section of this guided learning sequence repeats the above steps with skip-counts of 5. This could be taken as a separate teaching session.

9. Ask the learners to work out how much money you would need to give everyone in the room $5.

   "There are 14 of us in the room. How much money would I need if I were to give you each $5?"

10. Tell the learners to stand. Ask:

    "How can we count to solve the problem?"

11. If a learner suggests counting in ones, do that by asking each person in turn to count five and then sit down. For example, the first person counts "1, 2, 3, 4, 5" (then sits down), the second person counts "6, 7, 8, 9, 10" (then sits down) and so on to 70.

12. Hopefully someone will suggest counting in fives. Once more ask the learners to stand and take turns 'skip-counting' in fives. For example, the first learner says "5" (then sits down), the second leaner says "10" (then sits down) and so on to 70.

13. Ask the learners to use a different-coloured highlighter to shade in the skip-counts of 5 starting from 5 on the 10 x 10 grid.

14. When they have finished ask:

    "What do you notice about the numbers when you count (or skip-count) in fives?"

    - They all end in 5 or 0.
    - They form two columns on the chart.

15. How could you use this chart to help you work out 5 lots of 5. (You could use the chart to help keep track of the count of fives.)

16. Ask the learners to work in pairs to solve the following problems by skip-counting in fives. Encourage them to use the chart if they have difficulty keeping track of the number of fives they have counted.

    - 13 lots of 5
    - 6 lots of 5
    - 20 lots of 5
    - 9 lots of 5.

The final section in this guided learning sequence repeats the above steps with skip-counts of 3. This could be taken as a separate teaching session.

17. Ask the learners to suggest situations where you need to be able to count in threes. Use one of these suggestions as the context for the problems posed. For example, wheels on a tricycle.

18. Pose the problem:

    "How many wheels do you need for 14 tricycles?"

19. Tell the learners to stand, ask*:*

    "How can we count to solve the problem?"

20. Given the experience of the twos and fives, you would expect the learners to suggest counting in threes. If someone volunteers counting in ones, ask if they can think of a quicker way of counting.

21. Once more ask the learners to stand and take turns 'skip-counting' in threes. For example, the first learner says "3" (then sits down), the second leaner says "6" (then sits down) and so on to 42.

22. Give the learners a third coloured highlighter and ask them to shade in the skip-counts of 3 starting from 3 on the 10 x 10 grid.

continued...

23. When they have finished ask:

"What do you notice about the numbers when you skip-count in threes?" (They form diagonals on the chart. Some overlap with the twos or the fives.)

24. How could you use this chart to help you work out nine lots of three. (Use the chart to help keep track of the count of threes.)

25. Ask the learners to work in pairs to solve the following problems by skip-counting in threes. Encourage them to use the chart if they have difficulty keeping track of the number of threes they have counted.

    - 13 lots of 3
    - 7 lots of 3
    - 16 lots of 3
    - 4 lots of 3.

Follow-up activity

Give the learners a blank 10 x 10 grid and ask them to highlight the skip-counts of 4 starting from 4.
Ask them to write and solve skip-counts of fours problems, using the chart.

For example:

- 14 lots of 4 is 56
- 2 lots of 4 is 8
- 15 lots of 4 is 60.

Understanding multiplication

Multiplicative Strategies progression, 3rd step; Number Facts progression, 3rd step

The purpose of the activity

In this activity, the learners move from a 'repeated addition' model of multiplication to one where they use multiplication facts. They identify 'known' and 'unknown' multiplication facts and use already-known facts to develop quick recall of unknown facts.

The teaching points

- Using multiplication facts gives the same result as repeated addition but is more efficient – compare 6 x 9 = 54 with 9 + 9 + 9 + 9 + 9 + 9 = 54.
- Learners find out which multiplication facts they already know and derive unknown facts from known facts.
- Learners understand that 3 x 4 represents three groups of 4 and 4 x 3 represents four groups of 3 and that they both equal 12 (the commutative property of multiplication).
- Knowing the commutative property of multiplication reduces the number of multiplication facts to learn.
- Discuss with learners the occasions when they have derived facts from those they already know.

Resources

- Objects – paper clips, screws, etc.
- A set of multiplication fact cards for each learner (small cards with a fact on the front and the answer on the back).

The guided teaching and learning sequence

1. Write 3 x 4 on the board and ask the learners to represent this problem with objects. Possible representations are three groups of four and four groups of three. Ask the learners to share their representations with each other and notice any differences. Write 4 x 3 on the board and discuss the fact that it represents four groups of 3 while 3 x 4 represents three groups of 4.
2. Ask the learners to find out the total number of objects and share how they did it. Listen for counting 1, 2, 3, 4, etc. (counting all), 4 + 4 + 4 (repeated addition) and knowing 3 x 4 or 4 x 3 = 12 (basic facts). Ask the learners to consider which method is quickest – considering larger numbers may make the point clearer (6 x 9 = 54).
3. Encourage the learners to notice that while 3 x 4 and 4 x 3 mean different things, the total number of objects is the same.
4. Give each learner a set of multiplication fact cards and ask them to sort out the facts into two piles; one containing the facts they can recall the answer to quickly (in under 3 seconds) and the other containing the facts they can't. (Alternatively, the learners could work in pairs, taking turns to 'test' one another.)
5. Ask the learners to spread out the unknown fact cards and find pairs that have the same answer (for example 6 x 9 and 9 x 6). One of each pair can be removed from the 'unknown' fact pile.

continued...

6. Ask the learners to take an 'unknown' fact and share how they might work it out from a known fact.

   For example:

   If 4 x 7 is unknown but 2 x 7 = 14 is known, then 4 x 7 = 28 because it is 2 x 2 x 7.

   If 6 x 9 is unknown but 6 x 10 = 60 is known then 6 x 9 = 56 because it is 6 x 10 – 6 x 1.

   Explain to the learners that the aim is to eventually have all facts in their known pile. Have the learners store their two piles of cards in two envelopes so that they can be used in the "Deriving multiplication and division facts" activity to follow.

Follow-up activity

Ask the learners to find a known fact card from which they can derive the answer to an unknown fact card for each of their unknown facts. Ask them to explain how they solved the problem to another learner.

Deriving multiplication and division facts

Multiplicative Strategies progression, 3rd step; Number Facts progression, 3rd step

The purpose of the activity

In this activity, the learners extend their repertoire of multiplication and division facts and gain an understanding of the multiplication and division facts they need to practise for quick recall.

Traditionally, multiplication tables have been taught by rote, without an emphasis on learner understanding. This activity, however, follows international best practice and promotes an understanding of multiplication and division facts by using already-known facts to derive unknown facts. This activity builds on the 'Understanding multiplication' activity on page 35.

The teaching points

- The inverse of multiplication is division.
- The multiplication 'basic' facts are the facts made from the digits 0 to 9.
- Any number multiplied by 0 has an answer of 0.
- Any number multiplied by one has the original number as the answer.
- Multiplication and division facts can be directly connected. For example, 6 x 7 = 42, 7 x 6 = 42, 42 ÷ 7 = 6 and 42 ÷ 6 = 7 are connected facts.
- Unknown multiplication and division facts can be derived from known facts. For example if 6 x 7 = 42 then 7 x 7 = 42 + 7 = 49.
- Discuss with the learners the importance of being able to recall the basic multiplication and division facts. These facts are the 'building blocks' for estimation and more complex calculations.

Resources

- A set of multiplication and division fact cards, preferably one set for each learner.

The guided teaching and learning sequence

1. Write 8 x 9 = on the board.
2. Ask for a volunteer to give you the answer and record 72 on the board.
3. Ask the learners to think about how they might work out 8 x 9 if they had forgotten (or didn't know) it automatically.
4. Let volunteers share their strategies for working out 8 x 9.

   "I know that 8 x 10 is 80 and 8 less is 72."
   "I know 4 x 9 is 36, and I double it to get 72."
   "I know 8 x 8 is 64 and 8 more is 72."
   "I know the nines pattern." (18, 27, 36 ...)
5. Encourage the learners to notice that you can learn unknown facts by deriving (or figuring) them from already known facts.
6. Write 72 ÷ 8 = on the board and ask the learners if they know the answer. Encourage them to notice that this problem is directly connected to 8 x 9 = 72.
7. Ask the learners to state the other connected facts (72 ÷ 9 = 8 and 9 x 8 = 72.)
8. Ask the learners to think about the multiplication facts they have the most trouble with recalling. List these on the board.
9. Take one of the listed facts and ask the learners to think of all the ways they could work out that fact from other known multiplication or division facts. For example if 7 x 7 was recorded.

   "I know that 6 x 7 is 42, and 7 more is 49."
   "I know 7 x 8 is 56, and 7 less is 49."
   "I know 5 x 7 is 35, and 14 more is 49."

continued...

10. Ask:
    "If you know your 2 times facts, what other facts can you easily figure out?"
    Ensure all the learners understand the connections between the twos, fours and eights by looking at examples such as:
    3 x 2 = 6 so 3 x 4 = 12 and 3 x 8 = 24
    5 x 2 = 10 so 5 x 4 = 20 and 5 x 8 = 40
    Repeat this, using other linked facts. For example, derive the 6 facts from the 3 facts by doubling (for example, 3 x 5 = 15 so 6 x 5 = 30) and derive the 9 facts by subtracting from the 10 facts (for example, 10 x 3 = 30 so 9 x 3 = 30 – 3 = 27).

Follow-up activity

1. Ask the learners to spread out their known facts (from the "Understanding multiplication" activity above).
2. Next, ask them to take a fact from their unknown fact pile and see if it is closely connected to a known fact.
3. Ask the learners to explain how the unknown fact can be derived from the known fact. If the fact is not closely connected, ask the learner to take another fact from the unknown pile and repeat until they find a fact they can closely connect.
4. Ask the learners whether they think this fact now belongs in their known fact pile. If not, suggest they need to focus on practising/learning it.

Multiplication strategies

Multiplicative Strategies progression, 4th step

The purpose of the activity

In this activity, the learners develop mental strategies for solving multiplication problems with single-digit multipliers. Sticky notes are used to demonstrate strategies.

The teaching points

- There are a variety of mental strategies available for solving multiplication problems

  These include:

  - using tidy numbers with compensation (6 x 37 can be solved as (6 x 40) – (6 x 3))
  - place value partitioning (6 x 37 can be solved as (6 x 30) + (6 x 7))
  - deriving from known facts (8 x 25 = 200 because 4 x 25 = 100 is known and 2 x (4 x 25) = 200)
  - using equivalent expressions (8 x 25 = 200 because 4 x 50 = 200; 9 x 15 = 135 because 3 x 45 = 135).
- Different problems lend themselves to different strategies. Competent learners have a range of strategies and choose the most appropriate in a given situation.
- It is not intended that you name and 'teach' a range of strategies. This activity is designed to encourage the learners to explore and share strategies in order to increase their range.
- Using materials to demonstrate strategies helps develop understanding.
- Discuss some examples of everyday situations where the learners may need to use different strategies.

Resources

- Sticky notes.

The guided teaching and learning sequence

1. Use sticky notes to show six groups of 37 on the board and ask the learners to think of a way to find the total.

37	37	37	37	37	37

2. Ask the learners to share their strategies. If the strategies only include using a calculator or algorithm, ask the learners if they can think of any other ways. Model the strategies used on the board with sticky notes.

   Possible strategies include:

   a) Adding 3 to each group of 37 to make 6 forties. That makes 240. Take away the 6 threes (18). That makes 222. (Using tidy numbers with compensation.)

37 +3	37 +3	37 +3	37 +3	37 +3	37 +3

   b) Six thirties are 180. Six sevens are 42. 180 and 42 are 222. (Place value partitioning.)

30	30	30	30	30	30
7	7	7	7	7	7

   c) Using the algorithm. Ask the learners who use an algorithm to model it on the board and check their understanding of the underpinning place value ideas.

   For example, if they say 6 times 7 equals 42, carry the 4, 6 times 3 equals 18, add the 4, etc. check they understand that the 4 is actually 4 tens or 40, that 6 times 3 is actually 6 times 30 and equals 180, etc.

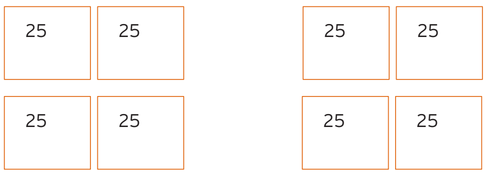
continued...

3. Use sticky notes to show 8 groups of 25 on the board and ask the learners to think of a way to find the total.

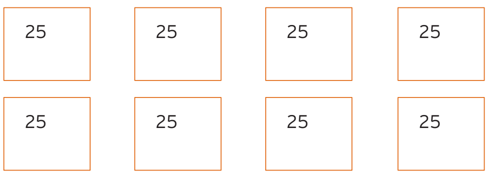
4. Ask the learners to share their strategies. Again, if strategies only include using a calculator or algorithm, ask the learners if they can think of any other ways. Model the strategy they use on the board with the sticky notes. (If they use strategies that have already been demonstrated, encourage them to think of further strategies.)

   Possible strategies include:

   a) "I know 4 groups of 25 makes 100 so 8 groups of 25 must be twice that. The total is 200."

   b) "Eight groups of 25 are the same as 4 groups of 50 because 2 lots of 25 are 50."

   c) Using the algorithm: ask the learners who use an algorithm to model it on the board and check their understanding of the underpinning place value ideas.

Follow-up activity

Write further problems on the board, for example, 5 x 68, 4 x 97, 9 x 44, 3 x 99, 7 x 26. Have the learners work in pairs and ask them to share their strategies for solving each problem. If they are unable to understand the strategy their partner uses, ask the partner to demonstrate it with the sticky notes.

When the learners appear comfortable with sharing and understanding strategies without using sticky notes, challenge them by introducing larger numbers, for example, 5 x 999, 7 x 306, 4 x 275, 8 x 179.

Division strategies

Multiplicative Strategies progression, 4th step

The purpose of the activity

In this activity, the learners develop mental strategies for solving division problems with single-digit divisors. Materials are used to demonstrate strategies.

The teaching points

- There are a variety of mental strategies available for solving division problems

  These include:

  - using tidy numbers with compensation (72 ÷ 4 can be solved as (80 ÷ 4) – 2)
  - deriving from known facts (72 ÷ 4 = can be solved by 76 ÷ 2 = 36 (known fact) and 36 ÷ 2 = 18 (known fact) because dividing by 4 is the same as dividing by 2 twice)
  - using reversibility (72 ÷ 4 can be solved by turning it into the multiplication 4 x ? = 72 and using multiplication strategies)
  - using equivalent expressions (72 ÷ 4 can be solved by 36 ÷ 2 (halving and halving))
  - 360 ÷ 5 can be solved by 720 ÷ 10 (doubling and doubling)

  Note: The answer to a division problem remains the same whenever you increase or decrease both the number to be divided and the divisor by the same factor (for example, multiply both by 10). This concept underpins the method commonly used, but poorly understood, for dividing with decimals. For example, 16 ÷ 0.4 is the same as 160 ÷ 4, where both numbers have been multiplied by 10.

- Different problems lend themselves to different strategies. Competent learners have a range of strategies and choose the most appropriate in a given situation.
- It is not intended that you name and 'teach' a range of strategies. This activity is designed for the learners to explore and share strategies in order to increase their range.
- Using materials to demonstrate strategies helps develop understanding.
- Discuss with the learners the strategies that are most familiar to them and how they can expand their repertoires.

Resources

- Sticky notes.
- Paper clips (lots).

The guided teaching and learning sequence

1. Tell the learners that you have 72 objects that need to be shared equally among 4 people and write 72 ÷ 4 on the board. Ask the learners to work out how many objects each person would get.
2. Ask the learners to share their strategies. If strategies only include using a calculator or algorithm, ask the learners if they can think of other ways of finding the answer. If the learners offer no strategies, prompt them by asking, for example, "What happens if I share 80 objects between 4 people ... and can I use this to help solve sharing 72 objects between 4 people?"

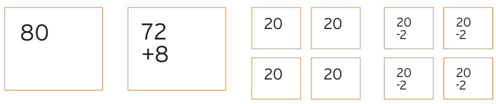
   Model the strategies discussed with paper clips or sticky notes.

continued...

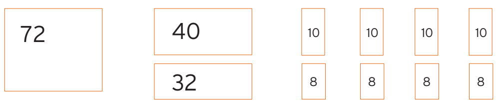
Possible strategies include:

a) Adding 8 to the 72 gives 80 objects, and if you share 80 equally among 4 people, each gets 20. But I 'shared' out 8 too many, so each person has 2 too many. Each person should have only 18. (Using tidy numbers with compensation or deriving from known facts.)

Model this by showing with sticky notes that the 8 added to give 80 shared equally between 4 people results in each person having an extra 2.



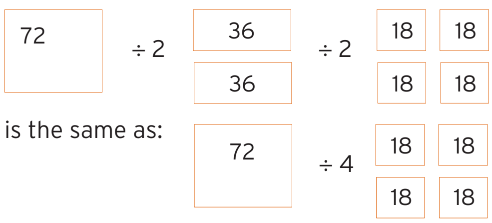
b) Saying: "4 times what equals 72? I know 4 times 10 equals 40 and I have 32 still to share out. I know 4 times 8 equals 32. Each person gets 18."



Another possibility is: "I know 4 times 15 equals 60, and I have 12 still to share out. I know 4 times 3 equals 12. Each person gets 18." (Using reversibility and place value partitioning.)

c) "Dividing by 4 is the same as dividing by 2 twice so 72 ÷ 2 = 36 and 36 ÷ 2 = 18." (Deriving from known facts.)

Model this, using sticky notes.



d) 72 is twice 36, and 4 is twice 2. I know if 36 is shared out between 2 people each gets 18, so if 72 is shared out between 4 people each will get 18.

In division, if each number is increased or decreased by the same factor, the answer will be the same. 72 ÷ 4 = 36 ÷ 2 = 18.

3. Model this by asking the learners to share 72 paper clips equally between 4 people and count how many each person gets. Ask the learners to halve the number of paper clips (36) and halve the number of people (2). Ask the learners to share 36 paper clips between 2 people and count how many each person gets. Emphasise that when 72 paper clips are shared out between 4 people and 36 between 2 people, each person gets the same amount.

4. Reinforce this concept by posing further problems.

   "Is 12 ÷ 6 the same as 6 ÷ 2?"
   "Is 90 ÷ 6 the same as 45 ÷ 3?"
   "Is 12 ÷ 6 the same as 6 ÷ 3?"

   If necessary, encourage the learners to show with paper clips that the answer to the original problem and the halved problem(s) is the same.

5. Ask the learners to investigate, with paper clips if necessary, whether the answer is the same if both numbers are doubled.

   "Is 15 ÷ 3 the same as 30 ÷ 6 and 60 ÷ 12?"

   Ask how doubling both numbers could be used to solve problems such as 360 ÷ 5.

Follow-up activity

Write further problems on the board, for example 120 ÷ 8, 78 ÷ 6, 171 ÷ 9, 240 ÷ 5. Have the learners work in pairs and ask them to share their strategies for solving each problem. If they are unable to understand the strategy their partner is using, ask the partner to demonstrate it with materials.

Multiplying options

Multiplicative Strategies progression, 5th step

The purpose of the activity

In this activity, the learners use strategies, traditional written methods and calculators to solve multiplication problems. The aim of exploring the three computation approaches is to encourage the learners to anticipate from the complexity and structure of a problem the approach that best suits them.

The learners need to be familiar with the concepts addressed in the "Multiplication strategies" activity on page 39 before starting this activity.

The teaching points

- Different multiplication problems lend themselves to different mental strategies, and competent learners have a range of strategies to choose from.
- There are three main computation approaches: using strategies to calculate a problem mentally, using traditional written methods (algorithms) and using calculators. The complexity of the problem determines the most effective and efficient computation approach.
- There are significant differences between mental strategies and traditional algorithms.
  - Mental strategies are number oriented rather than digit oriented. This means that the value of the number is considered rather than just the digit. Using the traditional algorithm, the learner solves 45 x 6 by thinking of 6 x 5 and 6 x 4 rather than 40 x 6 and 5 x 6.
  - Mental strategies tend to consider the largest value of the number first, while traditional algorithms start on the rightmost digits, which are the smallest in value.
  - Mental strategies are flexible and change with the numbers involved in order to make the computation easier. With the traditional algorithm, the same rule is used on every problem.
- Irrespective of the computation approach used, it is important that the learners are able to judge the reasonableness of their answer in relation to the problem posed.
- Discuss with the learners relevant or authentic situations where multiplication is used (for example, area measurements).

Resources

- Empty number lines.
- Calculators.

The guided teaching and learning sequence

1. Write the following problem on the board: 39 x 21 =
2. Discuss with the learners possible situations where they might need to do a calculation like that. For example: A set menu at the Blue Cheese Restaurant costs $39 per head. How much will it cost to take 21 people to dinner at the restaurant?
3. Discuss with the learners the three computation approaches that can be used to solve the problem:
   - using a calculator
   - using a written method
   - using a strategy.
4. Ask the learners to choose one of the options and solve the problem, giving them time to do this. Then ask them to share their solution and how they solved it with another learner. Tell them they also have to be able to explain why the answer they obtained is reasonable (or makes sense) in relation to the problem.

continued...

5. Ask the learners to indicate (with a show of hands) which of the three computation approaches they used.

6. Ask for a volunteer who used a calculator to solve the problem:

   "Why did you choose to use a calculator?"
   "Why is your answer reasonable?"

   For example, a learner could explain that 819 is reasonable as 39 x 21 is similar to 40 x 20, which equals 800.

7. Ask for a volunteer who used a written method to solve the problem.

   "Why did you use that method?"
   "Show us what you did on the board."

```
   39
 x 21
 ----
   39
  780
 ----
  819
```

   Notice if the learner has explained the algorithm using digits or the value of the numbers (for example, 2 x 3 = 6 or 20 x 30 =600).

   "How did you know that your answer was reasonable?"

   If the learner used the value of the numbers, their explanation of the algorithm might include an explanation of the reasonableness of the answer. If they used a digits-only explanation of the algorithm, encourage them to explain why 819 is reasonable as an answer to 39 x 21.

8. Ask for a volunteer who used a mental strategy to solve the problem.

   "Why did you decide to use a strategy?"
   "Explain what you did to work out the answer."
   "How did you know that your answer was reasonable?"

   Using place value partitions:
   (39 x 20) + (39 x 1) = 780 + 39

   or

   (30 x 21) + (9 x 21) = 630 + (9 x 20) + (9 x 1) = 630 + 180 + 9

   Using tidy numbers with compensation:
   (21 x 40) – 21 = 840 – 21

9. Pose another problem: 120 x 225 =

10. This time, ask the learners to work in pairs to solve the problem, using the three different approaches. As the learners solve the problem, discuss with them their preferred approach.

11. Give the learners the following problems on a sheet of paper. Without actually solving the problems, ask the learners to look at each problem and write down which approach they think they would prefer to use to solve the problem.

    Which approach will you use?

    a. 215 x 101

    b. 376 x 58

    c. 8576 x 99

    d. 780 x 450

12. As a group discuss which problems seem best suited to mental strategies and which they would prefer to solve with a calculator or written method. The learners may suggest that 8,576 x 99 could be solved by 8,576 x 100 – 8,576. However, it is important then to ensure that the learners consider whether 857,600 – 8,576 is a computation that they can do mentally.

This activity could be followed by or combined with the "Dividing options" activity on page 46.

Follow-up activity

Give the learners cards with one multiplication problem on each card. Have the learners work in pairs to select a computation approach, solve the problem and explain the reasonableness of their answer.

Dividing options

Multiplicative Strategies progression, 5th step

The purpose of the activity

In this activity, the learners use strategies, traditional written methods and calculators to solve division problems. The aim of exploring the three computation approaches is to encourage the learners to anticipate from the complexity and structure of a problem the approach that best suits them.

The learners need to be familiar with the concepts addressed in the "Division strategies" activity above before starting this activity. This activity could follow, or be completed in combination with, the "Multiplying options" activity on page 43.

The teaching points

- Different division problems lend themselves to different mental strategies, and competent learners have a range of strategies to choose from.
- There are three main computation approaches: using strategies to calculate a problem mentally, using traditional written methods (algorithms) and using calculators. The complexity of the problem determines the most effective and efficient computation approach.
- There are significant differences between mental strategies and traditional algorithms.
    - Mental strategies are number oriented rather than digit oriented. This means that the value of the number is considered rather than just the digit. Using the traditional algorithm, the learner solves 639 ÷ 3 by thinking of 6 ÷ 3 rather than 600 ÷ 3.
    - Mental strategies are flexible and change with the numbers involved in order to make the computation easier. With the traditional algorithm, the same rule is used on every problem.
- Irrespective of the computation approach used, it is important that the learners are able to judge the reasonableness of their answer in relation to the problem posed.
- Discuss with the learners relevant or authentic situations where division is used (for example, sharing a restaurant account).

Resources

- Calculators.

The guided teaching and learning sequence

1. Write the following problem on the board:
   534 ÷ 6 =
2. Discuss with the learners possible situations where they might need to solve a calculation like that. For example: The accommodation account for the weekend for 6 people at the Lazy Days Hotel came to $534. How much does each person need to pay (assuming they are paying equal amounts)?
3. Discuss with the learners the three computation approaches that can be used to solve the problem:
    - using a calculator
    - using a written method
    - using a strategy.
4. Ask the learners to choose one of the options and solve the problem giving them time to do this. Then ask them to share their solution and how they solved it with another learner. Tell them they also have to be able to explain why the answer they obtained is reasonable (or makes sense) in relation to the problem.

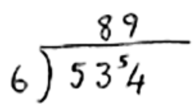
5. Ask the learners to indicate (with a show of hands) which of the three approaches they used.

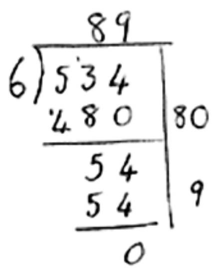
6. Ask for a volunteer who used a calculator to solve the problem.

   "Why did you choose to use a calculator?"
   "Why is your answer reasonable?"
   For example, a learner could explain that 89 is reasonable as 90 x 6 = 540.

7. Ask for a volunteer who used a written method to solve the problem.

   "Why did you use that method?"
   "Show us what you did on the board."





   Notice if the learner has explained the algorithm using digits (the first example) or the value of the numbers (the second example).

   "How did you know that your answer was reasonable?"

   If the learner used the value of the numbers, the explanation of the algorithm might include an explanation of the reasonableness of the answer. If they used a digits-only explanation of the algorithm, encourage them to explain why 89 is reasonable as an answer to 534 ÷ 6.

8. Ask for a volunteer who used a mental strategy to solve the problem.

   "Why did you decide to use a strategy?"
   "Explain what you did to work out the answer."
   "How did you know that your answer was reasonable?"

Using tidy numbers with compensation:
Round 534 to 600
600 ÷ 6 = 100
less 60 (10 lots of 6) = 540
less 6 (1 lot of 6) = 534
so 100 - 10 - 1 = 89 lots of 6
or
Round 534 to 540
6 x 90 = 540
so 6 x 89 = 534

9. Pose another problem: 12,342 ÷ 44 =

10. Discuss with the learners which of the three approaches they consider most efficient for solving this problem. Hopefully the learners will agree that using a calculator is the most efficient as it is quick and likely to be the most accurate. Try to avoid getting into a debate with the learners about the availability of calculators, explaining that in most situations where you need to calculate complex, accurate problems, both pen and paper and calculators will generally be available.

11. Ask the learners to work in pairs, using a calculator to answer the problem (12,320 ÷ 44 = 280). Also ask them to decide how they know if the answer obtained is reasonable.

12. Ask the learners to share the ways they considered that the answer was reasonable.

    Possible explanations include:
    Using rounding: 12,000 ÷ 40 = 300, which is close to 280.
    Using doubling: 280 x 100 = 28,000, which is 2-and-a-bit times larger than 12,340.

continued...

13. Give the learners the following problems on a sheet of paper. Without actually solving the problems, ask the learners to look at each problem and to write down which approach they would choose to solve the problem.

> Which approach will you use?
>
> a. 1,764 ÷ 18
>
> b. 1,800÷ 15
>
> c. 686 ÷ 7
>
> d. 704 ÷ 8
>
> e. 2,240 ÷ 20

14. As a class, discuss which problems are ones that seem best suited to mental strategies and which the learners would prefer to solve with a calculator or written method.

Follow-up activity

Give the learners cards with one division problem on each card. Have the learners work in pairs to select a computation approach, solve the problem and explain the reasonableness of their answer.

Multiplying with decimals

Multiplicative Strategies progression, 6th step

The purpose of the activity

In this activity, the learners use estimation strategies and calculators to solve multiplication problems involving decimals.

The teaching points

- The multiplication of two numbers results in the same digits regardless of the decimal point. For example, 24 x 59 = 1,416, 2.4 x 5.9 = 14.16, 24 x 0.59 = 14.16. Consequently there is no need to develop new approaches for multiplying decimals because the whole-number approaches apply and estimation can be used to work out where the decimal point should be placed.
- The use of estimation as a method for deciding on the position of the decimal is more difficult when the numbers are smaller. For example, knowing that 24 x 59 = 1,416 still does not make it straightforward to work out the answer to 0.0024 x 0.00059.
- It is as important to develop number sense with decimals as it is with whole numbers. While counting decimal points in multiplication problems works as a rule and has a conceptual rationale, it does not develop number sense.
- The decimal point is a convention that indicates the units, place. The role of the decimal point is to indicate the units, or ones, place in a number, and it does that by sitting immediately to the right of that place. Consequently, the decimal point also works to separate the unit (on the left) from parts of the unit (on the right).
- Irrespective of the computation approach used, it is important learners are able to judge the reasonableness of their answer in relation to the problem posed.
- When precision is important and the computation is difficult, then calculators and spreadsheets should be used.
- Discuss with the learners relevant or authentic situations where multiplication of decimals is used (for example, area measurements).

Resources

- Calculators.

The guided teaching and learning sequence

1. Write the following problem on the board: 25 x 0.33 =
2. Discuss with the learners possible situations where they might need to do a calculation like that. For example: Each can of coke holds 0.33 litres (330 millilitres). How many litres of coke are there in 25 cans?
3. Write the following estimates (20, 10, 5, 8) on the board and ask the learners to think about which is a reasonable estimate and which aren't reasonable. Discuss reasons for and against each estimate. Possible reasons include:
   - 0.33 is $^1/_3$ and $^1/_3$ of 25 is a bit more than 8 so 8 is the best estimate.
   - 25 x 3 is 75 so 25 x .3 = 7.5 so 8 is close to that.
4. Once 8 is established as the best estimate, tell the learners to use one of the following three computation approaches to determine the exact answer:
   - using a calculator
   - using written method
   - using a strategy.
5. Ask the learners to share their solution and how they solved the problem with another learner.
6. Ask the learners to indicate (with a show of hands) which of the three approaches they used.

continued...

7. Ask for volunteers to share the approach they used and their reasons for selecting that approach. Ensure the volunteers are able to explain their answer in relation to the agreed estimate of 8.

   "How did you know the decimal point went between the 8 and the 25?"

In the remainder of this activity, we focus on using estimation to work out where to place the decimal point in an answer.

8. Ask the learners to calculate the answer to 24 x 59. When they have agreed that 24 x 59 = 1,416, write the equation on the board.

24 X 59 = 1,416

9. Below this, write the following problems, telling the learners they are only able to use that first result of 1,416 to work out the exact answer to the problems.
   - 0.24 x 59
   - 2.4 x 5.9
   - 24 x 0.59
   - 0.24 x 0.59
   - 2.4 x 0.059

10. Ask the learners to work in pairs to write down their rationale for each answer. As the learners work on the problems, encourage them to think about the 'size' of the numbers involved. For example, 24 x 0.59 is close to a half of 24 so it makes sense to put the decimal point after 14. The learners could also reason that 1.416 is too small and 141.6 is too large.

11. Choose a couple of the problems to discuss as a class.

12. Give pairs of learners a calculator to compute the following products. Ask them to write a rationale for why the answer makes sense in terms of the numbers that were multiplied together.
    - 623.1 x 0.5 (311.55 makes sense as 300 is $^1/_2$ or 0.5 of 600).
    - 5.1666 x 5 = 25.833 (25.833 makes sense as 5 x 5 = 25).

Follow-up activity

Ask the learners to use 56 x 45 = 2,520 to give exact answers to the following problems. Ask them to write down or be prepared to explain the position of the decimal point in each answer.

56 x 45 = 2,520 5.6 x 4.5 = 0.56 x 0.45 = 56 x 0.45 = 560 x 0.045 =

Dividing with decimals

Multiplicative Strategies progression, 6th step

The purpose of the activity

In this activity, the learners use estimation strategies and calculators to solve division problems involving decimals.

The teaching points

- The division of two numbers results in the same digits regardless of the decimal point. For example, 754 ÷ 13 = 58, 75.4 ÷ 1.3 = 58, 7.54 ÷ 1.3 = 5.8, 0.754 ÷ 0.13 = 5.8, 7.54 ÷ 13. = 0.58. Consequently, there is no need to develop new approaches for dividing decimals as the whole number approaches apply and estimation can be used to work out where the decimal point should be placed.
- The use of estimation as a method for deciding on the position of the decimal is more difficult when the numbers are smaller. For example, knowing that 754 ÷ 13 = 58 still does not make it straightforward to work out the answer to 0.000754 ÷ 0.013.
- When precision is important and the computation is difficult, then calculators and spreadsheets should be used.
- The decimal point is a convention that indicates the units, place. The role of the decimal point is to indicate the units, or ones, place in a number, and it does that by sitting immediately to the right of that unit place. Consequently the decimal point also works to separate the unit (on the left) from parts of the unit (on the right).
- Irrespective of the computation approach used, it is important that learners are able to judge the reasonableness of their answer in relation to the problem posed.
- Discuss with the learners relevant or authentic situations where division of decimals is used (for example, rate problems).

Resources

- Calculators.

The guided teaching and learning sequence

1. Pose the following problem to the class:
   The distance from Dunedin to Queenstown is 287 kilometres. If it takes 3.5 hours to travel between the two cities, what is the average speed in kilometres per hour?
2. Discuss with the learners how they might estimate the answer to the problem. Share and record possibilities on the board.
   - 3.5 x 100 = 350, so maybe about 80 or 90 km/hr.
   - 4 x 80 = 320, so somewhere close to 80 km/hr.
   - 300 ÷ 3 = 100 and 300 ÷ 4 is a bit more than 70 km/hr.
3. Once somewhere between 80 and 90 km/hr has been established as an estimate, ask the learners to compute the exact answer. Discuss with the learners the options they have for the computation:
   - using a calculator
   - using written method
   - using a strategy.
4. Ask the learners to share their solution and how they solved it with a learner who used a different approach.

continued...

The remainder of this sequence focuses on using estimation to work out where to place the decimal point in an answer.

5. Write 287 ÷ 3.5 = 82 on the board.
6. Below this write the following problems, telling the learners they are only able to use that earlier result to work out the exact answer to the problems.

287 ÷ 3.5 = 82
28.7 ÷ 3.5 =
2.87 ÷ 0.35 =
0.287 ÷ 3.5 =
28.7 ÷ 35 =

7. Ask the learners to work in pairs to write down their rationale for each answer. As the learners work on the problems, encourage them to think about the 'size' of the numbers involved. For example, 28.7 ÷ 35 is going to give an answer less than 1 because 35 is larger than 28. The learners could also reason that it makes sense to put the decimal point before 82 (0.82) as there are not 8 lots of 35 in 28 and 0.082 lots of 35 is not close to 28.
8. Choose a couple of the problems to discuss as a class.
9. Give pairs of learners a calculator to compute the following quotients. Ask them to write a rationale for why the answer makes sense in terms of the numbers that were divided.
   - 63 ÷ 4.2 =
   - 7.446 ÷ 14.6 =
   - 27 ÷ 0.45 =

Follow-up activity

Ask the learners to use 156 ÷ 8 = 19.5 to give exact answers to the following problems. Ask them to write down or be prepared to explain the position of the decimal point in each answer.

156 ÷ 8 = 19.5
15.6 ÷ 0.8 =
0.156 ÷ 8 =
156 ÷ 0.8 =
0.156 ÷ 0.08 =

Ratios I

Proportional Reasoning Strategies progression, 4th step

The purpose of the activity

In this activity, the learners develop an understanding of simple ratios and learn to identify ratios for given quantities as well as quantities for given ratios.

The teaching points

- The learners understand relative quantities, for example, 'twice as many' is the same as two groups of the original number.
- The learners use ratio notation, for example, 2:1 is another way of writing 'twice as many'.
- Ratios, like fractions, are usually expressed in the simplest form, for example, 6:2 can be simplified to 3:1.
- The learners can name ratios for given quantities. The order of the numbers in the ratio must follow the order of the quantities expressed in the ratio. For example: If I have 8 apples and 4 bananas, the ratio of apples to bananas is 8:4, which can be simplified to 2:1.
- The learners can identify quantities for defined ratios. For example: If a recipe for jam requires 2 cups of fruit for every cup of sugar, possible amounts of fruit and sugar include 4 cups of fruit and 2 cups of sugar or 6 cups fruit and 3 cups of sugar.
- Discuss with the learners the ways in which they have used ratios in their work or home lives.

Resources

- 1-centimetre grid paper.
- Coloured pens.

The guided teaching and learning sequence

1. Ask the learners what the phrase "twice as many" means. Use these examples:

   "If I have $1 and you have twice as much money as me, how many dollars do you have?"
   "If I have 2 biscuits and you have twice as many biscuits as me, how many biscuits do you have?"
   "If I have 3 pencils and you have twice as many pencils as me, how many pencils do you have?"

   Check that the learners all understand that "twice as many" is a way of identifying the amount as 2 times the original or 2 groups of the original number.

2. Ask the learners what the phrases "3 times as many" or "4 times as many" mean. Uses these examples:

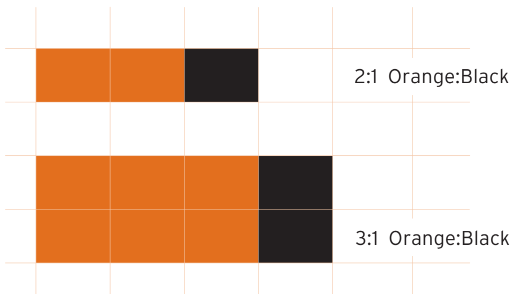
   "If you have 3 times as many cars as me, and I have 1 car, how many cars do you have?"
   "If you have 4 times as many pens as me, and I have 2 pens, how many pens do you have?"

   Check the learners all understand that "3 times as many" is a way of identifying the amount as 3 times the original and "4 times as many" is a way of identifying the amount as 4 times the original.

3. Write the ratio '2:1' on the board and ask:

   "What does a ratio of "2 to 1" mean?"

   Check the learners all understand that 2:1 is a way of representing twice as many.
   Ask the learners to draw some examples of simple ratios, using the grid paper and two different-coloured pens. For example:



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Check all the learners understand that the order of the colours in the ratio needs to be specified. Check also that they understand that 6:2 can be simplified to 3:1.

4. Ask the learners to simplify the following ratios:
   8:4 (2:1)
   3:6 (1:2)
   12:4 (3:1)

5. Tell the learners that some recipes use ratios to define quantities. Tell them a fruit salad calls for:

   - Bananas and apples in a ratio of 2:1
   - Oranges and apples in a ratio of 3:1

   Ask:

   "How many apples and oranges are there in a fruit salad that has 2 bananas?"
   (1 apple, 3 oranges)

   "How many bananas and apples are there in a fruit salad that has 9 oranges?"
   (6 bananas, 3 apples)

   "What other fruit salads could you make with these ratios?"

6. Ask:

   "If you had the following pieces of fruit to make a fruit salad, what ratios could you use to describe the quantities?"
   "3 bananas"
   "6 apples"
   "12 oranges"

   Check that all the learners are able to write ratios such as 3:6 bananas to apples, 3:12 bananas to oranges and 6:12 apples to oranges. Encourage the learners to simplify these ratios, for example 3:6 can be simplified to 1:2 and 3:12 can be simplified to 1:4.

Follow-up activity

Ask the learners to work in pairs to solve some of the following ratio problems:

- If a photo measures 4 centimetres by 9 centimetres and it is enlarged in a ratio of 2:1, what will the measurements of the enlarged photo be? (8 centimetres by 18 centimetres.)
- If Sally runs at a speed of 10 laps per minute, how far will she run in 5 minutes? If Jane runs one-and-a-half times as fast as Sally, how many laps can she do in 1 minute? How many laps can Jane run in 5 minutes? (Sally runs 50 laps in 5 minutes; Jane runs 15 laps in 1 minute and 75 laps in 5 minutes.)
- A jam recipe calls for fruit and sugar in a 2:1 ratio. List three different amounts of fruit and sugar that could be used. (2 cups of fruit and 1 cup of sugar, 4 cups of fruit and 2 cups of sugar, 6 cups of fruit and 3 cups of sugar.)
- Laura and Sophie are on a diet. The ratio between Laura's weight loss to Sophie's weight loss is 3:1. If Laura has lost 12 kilograms of weight, how much weight has Sophie lost?

Fractions of numbers I

Proportional Reasoning Strategies progression, 4th step

The purpose of the activity

In this activity, the learners develop an understanding of how to find a fraction of a whole number where the answer is also a whole number.

The teaching points

- The use of the language "3 parts of 7 equal parts" is the key to establishing meaning for the fraction $^{3}/_{7}$.
- The bottom number (denominator) of a fraction names the number of equal parts, while the top number (numerator) of a fraction tells how many of these parts.
- The learners will understand that if you multiply a number by a fraction greater than 1, you will get a result that is greater than the number ($4 \times {}^{4}/_{3} = {}^{16}/_{3} = 5\,{}^{1}/_{3}$). Alternatively if you multiply a number by a fraction less than 1, you will get a result that is smaller than the number ($4 \times {}^{2}/_{3} = {}^{8}/_{3} = 2\,{}^{2}/_{3}$)
- Discuss with the learners any misconceptions they may have had about the terms used.
- Discuss with the learners relevant or authentic situations where fractions are used.

Resources

- A whiteboard.

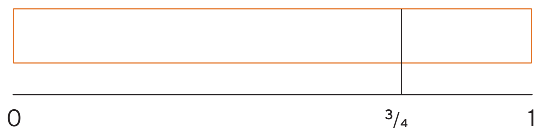
The guided teaching and learning sequence

1. Draw a bar (rectangle) on the board and ask a volunteer to place a mark to indicate where $^{3}/_{4}$ of the bar is. Ask:

   "How do you know that it is $^{3}/_{4}$?" (Listen to ensure the learners know that finding a quarter involves splitting the bar into 4 equal parts.)

2. Draw a line below the bar and on it record $^{3}/_{4}$.

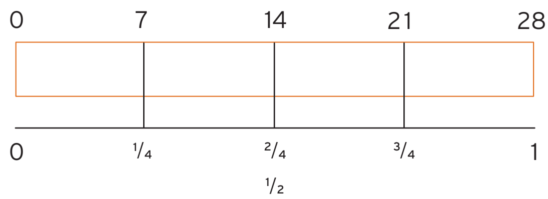
   Ask the learners to tell you what numbers to place at each end of the bar (0, 1). Discuss with the learners the fact that if the bar represents 1 unit, then its end points are 0 and 1. Record 0 and 1 on the bar.



3. Write 28 at the right end above the bar and ask the learners if they can work out what number is $^{3}/_{4}$ of 28. Check the learners understand that they must find the number that lies $^{3}/_{4}$ of the way along the bar. Ask the learners to share their answers and the ways they worked it out:

   "What is $^{3}/_{4}$ of 28?"
   "How did you work that out?"

4. Ensure that all the learners understand how they can use the unit fraction ($^{1}/_{4}$) of a number to help them work out a non-unit fraction ($^{3}/_{4}$) of that number. For example $^{3}/_{4}$ of 28 = $3 \times {}^{1}/_{4}$ of 28.



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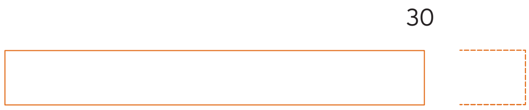
5. Depending on the learners' level of understanding, you may want to work through one or two more examples as a class before asking them to solve problems independently. Possible examples for further guided learning or independent work are:
    - $^4/_5$ x 30
    - $^5/_7$ x 42
    - $^2/_3$ x 18
    - $^4/_9$ x 27

The rest of this guided teaching and learning sequence repeats the above steps with fractions greater than 1. Depending on the learners' confidence with fractions less than 1, you may decide to complete the rest of this teaching and learning sequence in a future session.

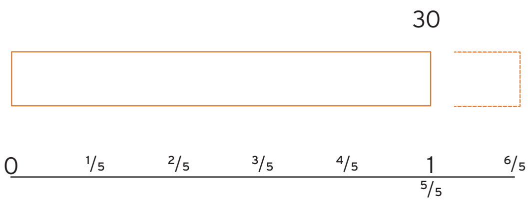
6. Write $^6/_5$ on the board and ask the learners to explain this fraction. Encourage them to note that the 5 tells that there are 5 equal parts (fifths) and that there are 6 of them. This means that the fraction is larger than 1.
7. Write $^6/_5$ x 30 on the board. Ask the learners to think about whether $^6/_5$ of 30 ($^6/_5$ x 30) will be a number that is greater than or less than 30.

    "Is $^6/_5$ of 30 larger or smaller than 30?"
    "Why is it larger than 30?"
8. Discuss the similarities between this problem and the problems the learners have solved with fractions less than 1.

    "Is this problem harder?"
    'Can we solve it the same way?"
    "What is another name for $^6/_5$?" (1 $^1/_5$)
9. The learners should be able to transfer their understanding from the easier problems to this problem and see that $^6/_5$ of 30 is the same as 6 x $^1/_5$ of 30 or 36. If not, work through the following representations.
10. Draw a bar on the board and label the end 30. Ask the learners to indicate where $^6/_5$ of the bar would be. From the previous discussion, the learners should understand why the answer is going to be larger than the 30.



11. Draw a line below the bar that extends at least to where the learners indicated $^6/_5$ of the bar would be and ask the learners to indicate where 0 and 1 are on the line. Ensure they understand that the 1 or unit in this problem is the 30.
12. Ask the learners if they think the previous mark for $^6/_5$ of 30 is in the correct position and why.



13. Once more, ensure all the learners understand how they can use the unit fraction ($^1/_5$) of a number to help them work out a non-unit fraction ($^6/_5$) of that number. For example, $^6/_5$ of 30 = 6 x $^1/_5$ of 30 = 36.
14. Depending on your learners' level of understanding, you may want to work through one or two more examples as a group before allowing the learners to work independently.
15. Pose problems for the learners to solve. Encourage them to share their solutions with others:
    - $^6/_4$ x 32
    - $^8/_3$ x 9
    - $^5/_2$ x 14
    - $^7/_{10}$ x 30

Finding fractions of numbers where the answer is also a fraction is covered in the "Fractions of numbers II" activity below.

Follow-up activity

If your learners have access to the Internet, they could practise the ideas developed in this activity by using the Fraction Bar learning object that is available from:

www.nzmaths.co.nz/LearningObjects/FractionBar/index.swf

This learning object poses problems that involve fractions less than 1 and problems that involve fractions less than 2. The learning object tracks the number of problems answered correctly, so you could challenge the learners to answer five problems that involve fractions less than 1 and then five problems that involve fractions less than 2.

If you are using the learning object, check the learners understand the bar represents the number that they are trying to find the fraction of and that the line below represents an 'amount' of that number. Learners at this stage should be familiar with the concept of a unit, and it is important that they realise that, for the purposes of this learning object, the number line from 0–1 represents the unit. Explain that for every problem, if they 'just know' the answer, they can always enter it, but otherwise, the steps they need to follow to solve the answer are similar to the ones presented in the guided lesson sequence above.

Ratios II

Proportional Reasoning Strategies progression, 5th step

The purpose of the activity

In this activity, the learners develop an understanding of more complex ratios, which cannot be simplified to a ratio that includes 1, for example 2:3. They explore the relationships between related ratios.

The teaching points

- Ratios, like fractions, are usually expressed in the simplest form. For example, 6:2 can be simplified to 3:1.
- The learners are able to name ratios for given quantities. The order of the numbers in the ratio must follow the order of the quantities expressed in the ratio. For example, if I have 8 apples and 4 bananas, the ratio of apples to bananas is 8:4, which can be simplified to 2:1.
- The learners can use ratio notation for more complex ratios. For example, the ratio 2:3 could be used to represent 4 apples and 6 bananas or 18 apples and 27 bananas.
- Equal ratios result from multiplication and division of existing ratios not from addition and subtraction. For example, multiplying both numbers in the ratio 2:3 by 2 gives an equal ratio of 4:6, but adding 2 to each number in the ratio 2:3 results in an unequal ratio of 4:5.
- The learners can find a ratio between two groups from two related ratios. For example, if the ratio between apples to oranges is 2:1 and the ratio between oranges and bananas is 2:1, the ratio between apples and bananas is 4:1.
- Discuss with the learners authentic situations where an understanding of ratios is important.

Resources

- Grid paper.
- Coloured pens.

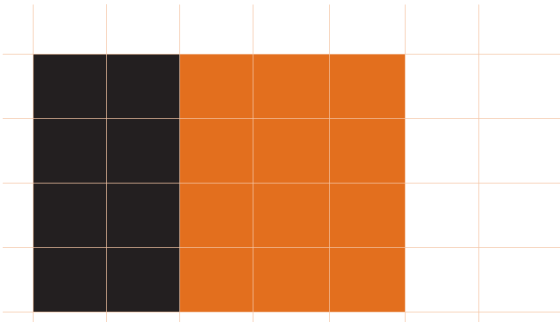
The guided teaching and learning sequence

1. Write the ratio '2:3' on the board and ask:

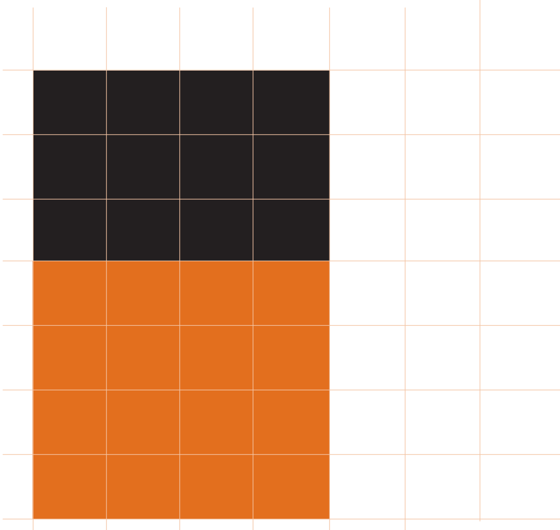
   "What does a ratio of "2 to 3" mean?"

   Check the learners all understand that 2:3 is a way of representing two groups that are made up of equal parts. One of the groups contains 2 equal parts, and the other group contains 3 equal parts.

   Ask the learners to draw some examples of more complex ratios, using grid paper and two different coloured pens. For example:



2:3 Black:Orange



3:4 Black:Orange

Check all the learners understand that the order of the colours in the ratio needs to be specified.

2. Tell the learners that some recipes use ratios to define quantities. Tell them that a fruit salad calls for:

   - bananas and apples in a ratio of 2:3
   - oranges and apples in a ratio of 5:3.

   Ask:

   "What numbers of bananas, apples and oranges could be used to make the fruit salad?"

   The simplest example would be 2 bananas, 3 apples and 5 oranges. Other possibilities include 4 bananas, 6 apples and 10 oranges or 6 bananas, 9 apples and 15 oranges.

3. Write the following recipes for fruit salad on the board:

Recipe 1	**Recipe 2**
Bananas and apples in a ratio of 4:7	Bananas and apples in a ratio of 8:14
Apples and oranges in a ratio of 7:10	Apples and oranges in a ratio of 3:5
Oranges and pears in a ratio of 2:3	Oranges and pears in a ratio of 6:9
Pears and kiwifruit in a ratio of 2:5	Pears and kiwifruit in a ratio of 5:8

   "Which of these fruits have equal ratios in the 2 recipes?" (bananas and apples, oranges and pears)
   "If you have 4 pears and 10 kiwifruit, which would be the best recipe to use?" (recipe 1)
   "If you have 12 apples and 20 oranges, which would be the best recipe to use?" (recipe 2)
   "If you have 4 oranges and 6 pears, which would be the best recipe to use?" (recipe 1)

   Check all the learners understand that equal ratios are the result of multiplying or dividing both numbers in the ratio by the same number.

4. Tell the learners that Sally has apples to oranges in the ratio 2:1 and oranges to bananas in the ratio 2:1. Ask:

   "If Sally has 1 banana, how many apples does she have?" (4)
   "If Sally has 3 bananas how many apples does she have?" (12)
   "If Sally has 20 apples, how many bananas does she have?" (5)
   "What is the simplest way of expressing the ratio between apples and bananas?" (4:1)

   Encourage the learners to use diagrams to answer these questions if necessary.

5. Ask the learners to solve the following problem:

   "Jane has apples, bananas and pears in a fruit salad. The ratio of apples to bananas is 2:1, and the ratio of bananas to pears is 3:1. What is the ratio of apples to pears?" (6:1)

6. Ask the learners to solve the following problems:

   Scott says the ratio of males to females in his class is 2:3 and there are 30 people in his class.

   "How many males are in Scott's class?" (12 males)
   "How many females are in Scott's class?" (18 females)

   Encourage the learners to share their ways for solving the problem with others. One solution is "I know that for every 5 learners, 2 are males and 3 are females. There are 6 lots of 5 learners in a class of 30. This means there are 6 x 2 = 12 males and 6 x 3 = 18 females."

continued...

Follow-up activity

Ask the learners to work together in pairs to solve some of the following ratio problems:

- A jam recipe calls for fruit and sugar in a 2:5 ratio. List three different amounts of fruit and sugar that could be used. (2 cups of fruit and 5 cups of sugar, 4 cups of fruit and 10 cups of sugar, 6 cups of fruit and 15 cups of sugar.)
- Alice and Jess are on a diet. The ratio between Alice's weight loss to Jess' weight loss is 2:3. If Alice has lost 12 kilograms of weight, how much weight has Jess lost? (18 kilograms)
- Find two equal ratios in each example:
    - 3:5, 5:7, 9:15 and 13:15
    - 2:3, 3:4, 12:16 and 7:8
    - 2:3, 3:4, 5:6 and 6:8
    - 3:4, 7:8, 14:16 and 6:7
    - 3:7, 6:14, 4:8 and 3:5.
- Cameron has apples, bananas and pears. The ratio of apples to bananas is 3:1 and the ratio of bananas to pears is 3:1. What is the ratio of apples to pears? (9:1)
- Jack has apples, bananas and pears. The ratio of apples to bananas is 4:1 and the ratio of bananas to pears is 2:1. What is the ratio of apples to pears? (8:1)
- Ann says the ratio of males to females in her class is 3:4, and there are 21 people in her class. How many males are in Ann's class? How many females are in Ann's class? (9 males and 12 females)

Fractions of numbers II

Proportional Reasoning Strategies progression, 5th step

The purpose of the activity

In this activity, the learners develop an understanding of how to find a fraction of a whole number where the answer may also be a fraction.

The teaching points

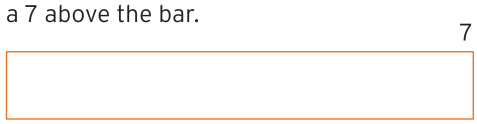
- The use of the language "3 parts of 7 equal parts" is the key to establishing meaning for the fraction $^{3}/_{7}$.
- The bottom number (denominator) of a fraction names the number of equal parts, while the top number (numerator) of a fraction tells how many of these parts.
- The learners will understand that if you multiply a number by a fraction greater than 1, you will get a result that is greater than the original number ($4 \times {^{4}/_{3}} = {^{16}/_{3}} = 5\,{^{1}/_{3}}$). Alternatively, if you multiply a number by a fraction less than 1, you will get a result that is smaller than the original number ($4 \times {^{2}/_{3}} = {^{8}/_{3}} = 2\,{^{2}/_{3}}$).
- Discuss with the learners ways to remember the names and functions of the fraction parts (denominator, numerator) and how to remember them.
- Discuss with the learners relevant and authentic situations where fractions are used.

Resources

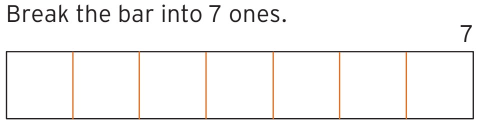
- A whiteboard.

The guided teaching and learning sequence

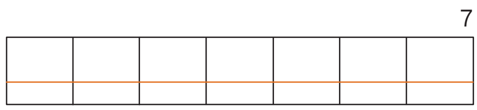
1. Draw a bar (rectangle) on the board and record a 7 above the bar.



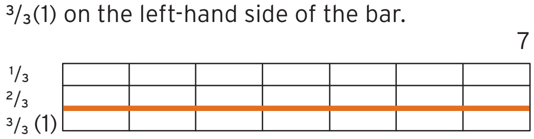
2. Break the bar into 7 ones.



3. Ask a volunteer to draw a line on the bar that shows where $^{2}/_{3}$ of 7 is. Note that there are two possibilities. One is a vertical line $^{2}/_{3}$ of the way along the bar which is somewhere between the fourth and fifth break. The second is a horizontal line that is $^{2}/_{3}$ of the way down the bar. Acknowledge both possibilities and then explain that you are going to use the horizontal line.



4. Ask the learners to show where $^{1}/_{3}$ of 7 would be as a horizontal line and record $^{1}/_{3}$ and $^{2}/_{3}$ and $^{3}/_{3}$(1) on the left-hand side of the bar.



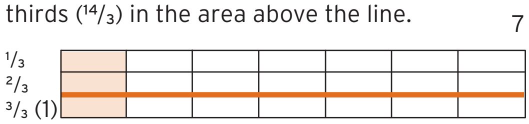
5. Ask:

   "What is $^{2}/_{3}$ of 7?" (Check that the learners understand that this is represented by the area above the $^{2}/_{3}$ line. Also check that the learners understand that this is the same as 2 lots of $^{1}/_{3}$ of 7).

   Ask the learners to share how they would work out 14 thirds ($^{14}/_{3}$) or 4 and $^{2}/_{3}$ of 7.

continued...

6. Check that all the learners understand that the 'smallest pieces' on the bar each represent $^{1}/_{3}$ of one whole. Shade in one of the pieces and explain that this is $^{1}/_{3}$ and that there are 14 thirds ($^{14}/_{3}$) in the area above the line.



7. Check all the learners understand how to convert 14 thirds ($^{14}/_{3}$) to 4 and 2 thirds ($4\,^{2}/_{3}$). Ask: "Why is 14 thirds equal to 4 and 2 thirds?" (Listen for explanations that use the understanding that 3 thirds are 1. For example, 3 thirds are 1 and therefore 14 thirds are 4 (ones) and 2 thirds or 14 thirds divided by 3 gives 4 ones with 2 thirds remaining.)

8. List the different ways of recording this on the board:

   - $^{2}/_{3} \times 7 = ^{14}/_{3}$
   - $^{2}/_{3} \times 7 = 4\,^{2}/_{3}$
   - Two-thirds of seven equals 14 thirds or four and two-thirds.

9. Depending on your learners' level of understanding, you may want to work through one or two more examples as a class before asking the learners to solve problems independently. Possible examples for further guided learning or independent work are:

   - $^{2}/_{5} \times 8$
   - $^{3}/_{4} \times 15$
   - $^{2}/_{3} \times 16$
   - $^{3}/_{7} \times 27$.

The rest of this guided teaching and learning sequence repeats the above steps with fractions greater than 1. Depending on your learners' confidence with fractions less than 1, you may decide to complete the rest of this teaching and learning sequence in a future session.

10. Write $^{5}/_{4}$ on the board and ask the learners to explain this fraction. Encourage them to note that the 4 shows that there are 4 equal parts (quarters) and that there are 5 of them. This means that the fraction is larger than 1.

11. Write $^{5}/_{4} \times 13$ on the board. Ask the learners to think about whether $^{5}/_{4}$ of 13 will be a number that is greater than or less than 13.

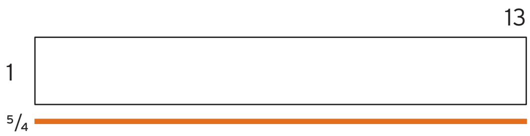
    "Is 5 quarters of 13 larger or smaller than 13?"
    "Why is it larger than 13?"

12. Discuss the similarities between this problem and the problems the learners have solved with fractions less than 1.

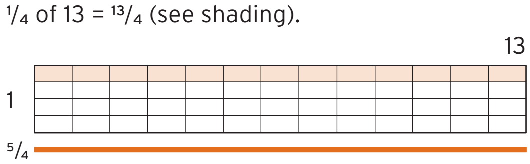
    "Is this problem harder?"
    "Can we solve it the same way?"
    "What is another name for 5 quarters?"
    ($1\,^{1}/_{4}$ or one and one-quarter)

13. The learners should be able to transfer their understanding from the easier problems to this problem and see that $^{5}/_{4}$ of 13 is the same as $5 \times ^{1}/_{4}$ of 13 or $^{65}/_{4}$ or $16\,^{1}/_{4}$. If not, work through the following representations.

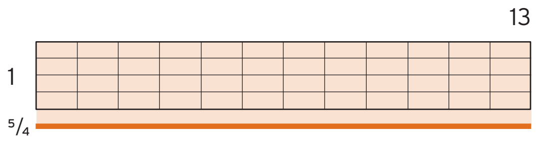
14. Draw a bar on the board and label the end 13. Ask the learners to indicate where $^{5}/_{4}$ of the bar would be and explain their reasoning. From the previous discussion, the learners should understand why the answer is going to be larger than the 13.



15. Once more, ensure that all the learners understand how they can use the unit fraction ($^{1}/_{4}$) of a number to help them work out a non-unit fraction ($^{5}/_{4}$) of that number. For example, $^{1}/_{4}$ of 13 = $^{13}/_{4}$ (see shading).



16. If the learners understand that $^1/_4$ of 13 is 13 quarters, then they should be able to understand that 5 quarters of 13 is 5 times this amount (13 x 5 = 65 quarters).



17. Depending on your learners' levels of understanding, you may want to work through one or two more examples as a group before allowing the learners to work independently.

18. Pose problems for the learners to solve. Encourage them to share their solutions with others.

    - $^5/_3$ x 7
    - $^3/_2$ x 21
    - $^4/_3$ x 16.

Follow-up activity

Pose problems for the learners to solve independently or in pairs.

- $^5/_4$ x 13
- $^7/_2$ x 9.

Rates and proportions

Proportional Reasoning Strategies progression, 6th step

The purpose of the activity

In this activity, the learners apply their understanding of ratios to rates and proportion problems. They learn that rates are examples of ratios where a comparison is made between quantities of different things and that solving proportional problems involves applying a known ratio to situations that are proportionally related and finding one of the measures when the other is given.

The teaching points

- A ratio can also be a rate. Part-to-whole and part-to-part ratios compare two quantities of the same thing. Rates on the other hand are examples of ratios where a comparison is made between quantities of different things. In rates, the measuring units are different or the quantities being compared are different and the rate is expressed as one quantity *per* the other quantity. For example, the value of food can be expressed as price per kilogram, fuel efficiency can be expressed as litres per 100 kilometres. Finding the unit rate is one way of solving proportional problems.
- Solving proportional problems involves applying a known ratio to situations that are proportionally related and finding one of the measures when the other is given. For example, if it takes 10 balls of wool to make 15 beanies, 6 beanies will take 4 balls of wool. In this example, the ratio of 2:3 (balls to beanies) can be applied to each situation.
- There are three main computation approaches: using strategies to calculate a problem mentally, using traditional written methods (algorithms) and using calculators. The complexity of the problem determines the most effective and efficient computation approach.
- Discuss with the learners relevant or authentic situations where rates are used (price per kilogram, kilometres per hour).

Resources

- Calculators.

The guided teaching and learning sequence

1. Write the following problem on the board.
   At the supermarket, you want to buy the best value fruit yoghurt. Assuming that you are not concerned about how much yoghurt you buy, which should you purchase?

	WEIGHT	PRICE
1	250 grams	$2.94
2	450 grams	$3.91
3	500 grams	$4.19
4	600 grams	$4.24
5	Packet of 4 (150 grams each)	$4.99

2. Discuss with the learners their understanding of the problem.

   "What does 'best value' mean?" (The lowest price for the same amount)

   "How many grams are in the packet of 4?"

3. Ask the learners to look at the list and see if there are any that are obviously not the best value.

   "Which options are definitely not the best value?"
   "How do you know?"

   Possible explanations include:

   - Option 1 is close to $3 for 250 grams, so at $6 for 500 grams, it is more expensive than options 3, 4 or 5.
   - Option 5 is 600 grams and is more expensive than the 600 grams of option 4.

4. Ask the learners for their ideas about how to compare options 2, 3 and 4. Hopefully someone will suggest that you can work out how much each costs as a rate (price per gram or per 100 grams or per kilogram) and then do a direct comparison. This approach can be referred to as a 'unit-rate' method of solving proportions.

5. Add a column to the table, agree on the rate (price per 100 grams) and then ask the learners (working individually or in pairs) to calculate the unit rate for options 2, 3 and 4.

	WEIGHT	PRICE	PRICE PER 100 GRAMS
1	250 grams	$2.94	X
2	450 grams	$3.91	
3	500 grams	$4.19	
4	600 grams	$4.24	
5	Packet of 4 (150 grams each)	$4.99	X

6. Remind the learners that they can choose to use a calculator, a written method (traditional algorithm) or mental strategy to work out the unit rate. Tell them you expect them to be able to explain the reasonableness of their answers irrespective of the approach they used.

7. Ask for volunteers to record one of the rates on the table and to explain to the class how they obtained it. Ask them to explain why they chose a particular approach and how they knew the answer made sense.

   "Why did you choose to use [approach]?"
   "Why is your answer reasonable or why does your answer make sense?"

8. Look at the table and discuss other ways that the rates could be expressed. Add these as columns to the table on the board, asking the learners to work out the new unit rates. Discuss with the learners the fact that rounding errors mean that conversions from the first rate will sometimes differ from calculations in relation to the original value. However, in relation to the context of this problem, all that was required was the 'best value' item rather than the exact unit rate for each item.

	WEIGHT	PRICE	$ PER 100G	$ PER KG	CENTS PER GRAM
1	250 grams	$2.94	X	X	X
2	450 grams	$3.91	$0.85	$8.50	
3	500 grams	$4.19	$0.84	$8.38	
4	600 grams	$4.24	$0.71	$7.10	
5	Packet of 4 (150 grams each)	$4.99	X	X	X

continued...

9. Pose the following problems for the learners to work on in pairs. As they work, discuss with them the computation approach they have selected and their explanation for the reasonableness of their answers.
    - Dan can run 5 kilometres in 16.3 minutes. If he keeps running at the same speed, how far can he run in 27 minutes?
    - Jack can run an 8-kilometre race in 37 minutes. If he runs at the same rate, how long should it take him to run a 5-kilometre race?
    - The cost of a box containing 48 chocolate bars is $45.00. What is the cost of 8 bars?

Follow-up activity

Ask the learners to write a proportion problem (similar to those shown above). Have them swap their problem with that of another learner, solve the new problem and then check one another's solutions.

Numbers to 100

Number Sequence progression, 2nd step

The purpose of the activity

In this activity, the learners focus on the order or sequence of numbers from 0 to 100. They learn to read and write numbers to 100 as well as position the numbers on an ‘empty’ number line.

The teaching points

- There are patterns to the way numbers are formed. For example, each decade has a pattern of the 1 to 9 sequence.
- An understanding of the number sequence is closely connected to an understanding of place value. The position of a digit in a number indicates what value that digit represents. For example, the 4 in 24 stands for 4 ones; the 4 in 45 represents 4 tens or 40.
- The way we say a number such as “sixty-three” is connected to the base-ten language of the number system.
- Discuss with the learners how counting by ones, tens and other groupings plays a key role in constructing understandings about the order and relative position of numbers.

Resources

- Empty number lines.

The guided teaching and learning sequence

1. Draw a number line on the board. Label the ends with a 0 and 100 respectively.

Ask: “Where would you place the number 70 on this number line?”

2. Get a volunteer to indicate the position and label it with 70. Ask them to explain why they positioned the number there. Encourage the learners to locate numbers with reference to benchmark numbers such as 50. Write down the word “seventy” with the 70 to reinforce the way it can be written and spoken.

3. Ask the learners to name the number that comes one before and one after 70. Use this as an opportunity to discuss the 1 to 9 sequence in each decade.
4. Repeat the process with other numbers.
5. Explain to the learners that you are going to write a number between 0 and 100 on a piece of paper. Their challenge is to guess the number by asking questions that you can answer with a yes or no. The aim is to guess the number with the least number of questions.
6. For example: You choose the number 78.

   “Is the number 56?” (No)
   “Is the number between 50 and 100?” (Yes)
   “Is the number between 50 and 80?” (Yes)
   “Is the number between 50 and 60?” (No)

   And so on until the number is guessed.
7. Ask a volunteer to write down a number and answer questions from the class until the number is guessed.
8. Alternatively, the activity could be played with pairs or small groups. As the groups or pairs play “guess the number”, ask them to work together to develop a strategy to guess the numbers using the least amount of questions.

continued...

9. Give groups or pairs the opportunities to challenge other groups. Conclude by sharing strategies for playing the game.

Follow-up activity

Ask the learners to position the number one larger and the number ten less than a given number on an empty number line. Also ask them to write the number in words.

- 40 (41, 30, forty)
- 15 (16, 5, fifteen)
- 79 (80, 69, seventy-nine).

Understanding fractions I

Ordering fractions with the same denominator ($^1/_3$, $^2/_3$, $^3/_3$, $^4/_3$)

Number Sequence progression, 3rd step

The purpose of the activity

In this activity, the learners develop an understanding of fractions by cutting, naming and ordering strips of paper.

The teaching points

- The learners understanding the names and symbols for fractions.
- The denominator (bottom number) in a fraction indicates the number of **equal** parts a whole is divided into.
- The numerator (top number) in a fraction indicates the number of those **equal** parts.
- Whole numbers can be written as fractions (for example $1 = {}^3/_3$ and $1{}^1/_3 = {}^4/_3$).
- Discuss with the learners the contexts in which they see or use fractions.

Resources

- Four (or more) strips of paper (card) for each learner approximately 5 centimetres by 25 centimetres. (It is useful, but not necessary, for each strip to be a different colour.)
- Scissors.

The guided teaching and learning sequence

1. Ask the learners to prepare a 'set of fraction strips' by:
   - keeping one strip as a single-unit strip
   - cutting one strip into two equal parts
   - cutting one strip into three equal parts
   - cutting one strip into four equal parts.
2. Discuss and ask the learners to write the symbol and name on the strip for each fraction type.
3. Encourage the learners to look for a relationship between the symbol and the fraction piece. Listen for the idea that the bottom number represents the number of pieces the strip is cut into and develop and emphasise the idea that the pieces must be equal. You could do this by cutting a strip into two unequal parts and asking if each is half. Introduce the name "denominator".
4. Ask the learners to work in groups. Ask one learner to put down a 'third' and a second learner to put one of their 'thirds' beside it. Discuss how many thirds are in the row and what symbol describes this. Continue with the learners adding 'thirds' to the row and recording the symbols ($^1/_3$, $^2/_3$, $^3/_3$, $^4/_3$). Encourage the learners to look for a relationship between the symbol and the fraction pieces. Listen for the idea that the top number of the fraction represents the number of equal parts. Introduce the name "numerator".
5. Ask the learners to place three 'thirds' in a row and think of another name for this. Listen for " $^3/_3$ is the same as 1" and ask the learners to check this against the single-unit strip. Repeat with four 'thirds' ($1{}^1/_3$), five 'thirds', etc.
6. Repeat the sequence from step 4 above, this time working with quarters.

Follow-up activity

Ask the learners to work in pairs to choose a fraction (for example, fifths or tenths) and to write down its sequence, giving equivalent names where possible (for example, $^5/_5 = 1$, $^6/_5 = 1{}^1/_5$).

Encourage the learners to keep the set of fraction cards for future lessons.

Understanding fractions II

Ordering unit fractions ($^1/_3$, $^2/_3$, $^3/_3$, $^4/_3$)

Number Sequence progression, 4th step

The purpose of the activity

In this activity, the learners order unit fractions using strips of paper. The learners need to be familiar with the concepts addressed in the "Understanding fractions 1" activity (ordering fractions with the same denominator) before they start this activity.

The teaching points

- For unit fractions, as the value of the denominator gets larger, the size of the fraction gets smaller.

  (Note: This is a difficult concept for learners because of the previous knowledge that, for whole numbers, the sequence 1, 2, 3, 4, 5, ... relates to an increase).
- Unit fractions are placed between 0 and 1 on the number line.
- Discuss with the learners what they already know and how they use what they know when solving a fraction problem.

Resources

- A set of fraction strips as for the "Understanding fractions 1" activity (ordering fractions with the same denominator).
- Cards: one with 0, one with 1 and the rest with one unit fraction (each learner needs one unit fraction card).

The guided teaching and learning sequence

1. Ask the learners to place the fraction strips for $^1/_2$, $^1/_3$ and $^1/_4$ in order from smallest to largest.
2. Ask the learners to identify which is bigger $^1/_2$ or $^1/_3$ and to describe how they know this. Encourage them to use their fraction strips if they are unable to answer this question. Listen for the idea that if a whole is divided into two equal parts, each part is bigger than if it is divided into three equal parts.
3. Repeat this question comparing $^1/_3$ and $^1/_4$.
4. Write $^1/_2$, $^1/_3$, $^1/_4$ on the board and ask the learners to look for a pattern between the denominator and the size of the fraction. Listen for the idea that as the denominator gets bigger, the fraction gets smaller.
5. Ask the learners to consider whether $^1/_5$ or $^1/_6$, $^1/_7$ or $^1/_9$, $^1/_{99}$ or $^1/_{100}$ is bigger and to describe how they know this.
6. Once the learners are familiar with the concept:
   - place the card labelled "0" at one end of the room
   - place the card labelled "1" at the other end of the room
   - give each learner a card with a different unit fraction on it and ask the learners to stand in the 'correct' place between 0 and 1.

Follow-up activity

The learners can place sets of unit fractions in order from smallest to largest.

Understanding fractions III

Fractions in division

Number Sequence progression, 4th step

The purpose of the activity

In this activity, the learners further develop their understanding of fractions by cutting and sharing strips of paper equally among a given number of people. The activities "Understanding fractions I and II" should be completed first.

The teaching points

- The learners will understand how to use fractions in division problems where the numbers do not divide evenly.
- The learners will understand the relationship between division and fractions – that the number to be shared out is represented by the numerator and the number of ways it is shared by the denominator. For example, if 3 strips are to be shared between 2 people, each person will get $^3/_2$ of a strip. (The learners may have already come across $3 \div 2$ written as $^3/_2$.)
- The learners will study alternative ways of writing fractions (for example, $^3/_2$ is the same as $1\,^1/_2$).
- Discuss with the learners other ways they have seen fractions written.
- Discuss with the learners the contexts in which they use fractions.

Resources

- Strips of paper (approx 5 centimetres by 25 centimetres).
- Scissors.

The guided teaching and learning sequence

1. Have the learners work in pairs and give each pair three strips of paper and ask them to share the strips equally between the two of them.
2. Record the different strategies the learners use to share the strips. Some may cut each strip in half so each person has $^3/_2$ strips, while others may give one strip to each person and cut the remaining strip in half so each has $1\,^1/_2$ strips.
3. Ask the learners:

   "Do these numbers both represent the same amount?" Explain why.
   "Can you see a relationship between the number of strips to be divided and the number of people they are to be divided between?"
4. Ask the pairs to join with another pair to form groups of 4 and predict how many strips each person would get if they shared 6 strips of paper equally between them. Ask the learners to share the strips out and check their predictions. Record the different strategies that the learners used ($^6/_4$, $1\,^2/_4$, $1\,^1/_2$) – suggest other possibilities if not all strategies are offered.
5. Again ask the learners:

   "Do these numbers all represent the same amount?" Explain why.
   "Can you see a relationship between the number of strips to be divided and the number of people they are to be divided between?"

continued...

6. Ask the class to regroup to form groups of 6 and have the groups predict how many strips each person would get if they shared 4 strips of paper equally between them. Ask the learners to share the strips out and check their predictions. Record the different strategies the learners used ($^4/_6$, $^2/_3$) – suggest other possible strategies if not all are offered. Check whether the relationship suggested in sequence 3 and 5 above ("between the number of strips to be divided and the number of people they are to be divided between") still holds.

Follow-up activity

Ask the learners to record the fractions for the following problems and discuss their answers with other learners.

- 7 people sharing 2 strips equally
- 2 people sharing 7 strips equally
- 15 people sharing 4 strips equally
- 12 people sharing 24 strips equally.

Introducing place value

Place Value progression, 2nd and 3rd step

The purpose of this activity

In this activity, the learners develop the understanding that our number system is based on the number 10. The place of a digit in a number indicates the size of that digit and the places increase by a factor of 10 as you move to the left and decrease by a factor of 10 as you move to the right.

The teaching points

- There is a system to our numbers that the learners need to understand. This system developed over time, and there are other number systems. A few interesting facts are as follows:
  - The first number system developed in Mesopotamia (now Iraq) approximately 4,800 years ago, based on 60. Our system of measuring time comes directly from that number system.
  - Not all number systems are place value systems. In the Roman number system, a number is often the first letter of the word it represents – some learners may be familiar with Roman numerals.
  - Base ten systems developed independently in China and India.
  - Our current system travelled from India through Arabia to Europe, arriving in Europe in the thirteenth century.
- The system we use is based on 10 and developed because we have 10 fingers. It requires the use of the digit 0 as a place holder. Once ten objects have been counted, a place holder is used to indicate the number of groups of ten (10). Once ten groups of 10 have been counted, a second place holder is used to indicate the number of groups of 10 tens (hundreds) (100), and so on.
- The value of the places increases to the left by a factor of 10 and decreases to the right by a factor of 10.
- The pattern of ones, tens, hundreds repeats for thousands and millions, and recognising this pattern helps with reading large numbers.

MILLIONS			THOUSANDS			ONES		
100s	10s	1s	100s	10s	1s	100s	10s	1s

- One billion is used to describe both one thousand million (1,000,000,000, 1×10^9 , United States of America) and one million million (1,000,000,000,000, 1×10^{12}, United Kingdom). Increasingly, the meaning of one billion as one thousand million is used in English-speaking countries.

Resources

- Calculators.

The guided teaching and learning sequence

1. Write 4**4** on the board and ask the learners whether both fours mean the same thing. Listen for the response that one 4 means 4 ones and the other 4 means 4 tens. Ask whether the learners agree that the place of the 4 indicates its value and reinforce that, in 4**4**, the bold 4 indicates 4 ones and the 4 on its left indicates 4 tens.
2. Write 444 on the board and ask the learners what the new four represents?
3. Draw a three-column place-value chart on the board and ask the learners what the relationship is between the value of the three places: ones, tens, hundreds.

HUNDREDS	TENS	ONES
4	4	4

Listen for and reinforce the response that each place increases by a factor of 10 as you move to the left.

continued...

4. Ask the learners why our number system is based around 10. If necessary, prompt the response that it is because we have 10 fingers. Model counting 12 objects on your fingers. Record the 1 group of 10 in the tens column before reusing your fingers to count the remaining 2. Record the 2 in the ones column.

HUNDREDS	TENS	ONES
	1	2

Ask what you would write in the place-value chart if you had only 10 objects. Discuss the use of 0 as a place holder.

5. Ask what happens when you have counted 9 groups of 10 and you add another group of 10. Listen for the response that you have 100, record this in the place-value chart and reinforce that 100 is the same as 10 groups of 10.

HUNDREDS	TENS	ONES
1	0	0

If necessary, the learners can count toothpicks into groups of 10 to develop a better understanding of this idea. Keep the groups in place with rubber bands.

6. At this point, you may wish to discuss the fact that not all number systems are based around 10 and perhaps talk about other number systems used around the world. The learners could do some research themselves into different number systems and their history. This is an opportunity for the learners to recognise that there is a cultural aspect to number and for you to value the different cultural backgrounds of your learners.

7. Draw the place-value chart below on the board.

						HUNDREDS	TENS	ONES

Ask the learners what value the place to the left of the 'hundreds' has. Listen for the response 'thousands' and ask the learners to confirm that this place is greater than the 'hundreds' place by a factor of 10 (10 x 100 = 1,000).

Repeat the questions:
"What value does the place on the left have?"
"Is it greater by a factor of 10?"

for each section, until the chart is complete.

100 MILLIONS	10 MILLIONS	1 MILLIONS	100 THOUSANDS	10 THOUSANDS	1 THOUSANDS	HUNDREDS	TENS	ONES

8. Ask the learners to look for and describe any pattern.

9. Redraw the chart.

MILLIONS			THOUSANDS			ONES		
HUNDREDS	TENS	ONES	HUNDREDS	TENS	ONES	HUNDREDS	TENS	ONES

10. Put numbers in the chart and ask:

Read this number aloud.
"What is the value of the 8?"
"What is the value of the 3?"
"What is the value of the 6?"

MILLIONS			THOUSANDS			ONES		
HUNDREDS	TENS	ONES	HUNDREDS	TENS	ONES	HUNDREDS	TENS	ONES
	2	6	7	8	9	1	4	3

Follow-up activity

Have the learners work in pairs with a calculator.

One of the pair keys a three-digit number into the calculator and asks the other to read it.

The first learner then asks the second learner to change a digit to another specified digit.

For example: A learner keys in 469 and says, "Read the number to me".

Then that same learner says, "Change the 4 into a 3". (It is necessary to subtract 100 on the calculator to change the 4 into a 3.)

And then they say, "Change the 6 to 7".
(It is necessary to add 10 on the calculator to change the 6 into a 7.)

Note: If the learners are operating above step 3 on the Place Value progression, they could work with larger numbers, for example, 25,482.

Whole number place value

Place Value progression, 3rd step

The purpose of the activity

In this activity, the learners extend their understanding of the place value of digits in a whole number by adding and subtracting 1, 10, 100 and 1,000 from a given four-digit whole number.

The teaching points

- The places increase by a factor of 10 (1, 10, 100, 1,000).
- There are challenges in dealing with the digit 9.
- The learners understand the use of 0 as a place holder.
- The learners can rename thousands as hundreds, hundreds as tens and tens as ones.
- Discuss with the learners relevant or authentic situations where the understanding of whole-number place value is necessary. For example, stocktaking, car sales with trade-in deals.

Resources

- Two dice for each pair of learners (wooden dice with blank faces are available from educational supply shops). One dice in each pair has the symbol for adding on three of its faces and subtracting on the other three (+, +, +, –, –, –). The other dice has the numbers 1, 10, 10, 100, 100 and 1,000 on its six faces.

The guided teaching and learning sequence

1. Write a four-digit number (not including the digits 0 or 9) on the board, for example, 3,467.
2. Ask a volunteer learner to roll the two dice.
3. Demonstrate how to write the resulting problem beside the four-digit number. (For example, if the dice show + and 100, record "3,467 + 100 = ?")
4. Ask the learners what the answer would be, have them explain their thinking and record "3,467 + 100 = 3,567".
5. Have another learner roll the dice again and repeat the process a few times, always using the previous answer as the new four-digit number to add to or subtract from.
   For example, since 3,467 + 100 = 3,567 was the first equation, the next might be 3,567 – 1000 = 2,567 and the following one might be 2,567 – 10 = 2557.
6. Repeat the whole activity with a different starting number.
7. Once the learners are familiar with the process, ask them to do the activity by themselves in pairs (selecting numbers that do not include the digits 0 and 9) and to share their results with another pair.

When the learners are using this process confidently and successfully, ask them to consider numbers that include the digits 9 and 0.

8. Write a problem with a number that includes the digit 9 on the board, for example, 4,975 + 100 = ? Ask the learners what the answer is and have them explain their thinking. Listen for and reinforce the idea that 10 hundreds is the same as 1 thousand so that when a place holding the digit 9 is increased by 1, the next (higher) place (on the left) also increases by 1.
9. Repeat with other numbers, for example 3,091 + 10 = ?, 4,309 + 1 = ?
10. Write a problem with a number including the digit 0 on the board, for example, 4,056 – 100 = ?

    Discuss the difference between 4,056 and 456 and the use of 0 as a place holder.

11. Ask the learners what the answer is and have them explain their thinking. Listen for and reinforce the idea that 4 thousand can be renamed as 40 hundred and that 40 hundred and 56 take away one hundred is 39 hundred and 56.
12. Repeat with other numbers, for example, 4,605 – 10.

Follow-up activity

Give the learners a set of numbers coupled with answers and ask them to work out what the dice must have rolled to come up with these answers.

For example, if the original number was 4,582, what must the dice have rolled in order to get:

- 4,482? (– and 100)
- 5,482? (+ and 1,000)
- 5,481? (– and 1).

Decimal number place value

Number Sequence progression, 5th step; Place Value progression, 5th step

The purpose of the activity

In this activity, the learners develop their understanding of the place value system to include the decimal numbers tenths, hundredths and thousandths. They learn to name any decimal number in tenths, hundredths and thousandths.

The teaching points

- The system for whole numbers, where each place to the right is smaller by a factor of 10, continues for the decimal numbers.

  For example:

  - The value of the first place to the right of 'one' is 'tenths' – 0.1 is $^1/_{10}$ of 1, 0.2 is $^2/_{10}$ of 1, etc
  - The value of the second place to the right of 'one' is $^1/_{10}$ of $^1/_{10}$ of 1 or $^1/_{100}$ of 1 – 0.01 is $^1/_{100}$, 0.23 is $^{23}/_{100}$ or $^2/_{10}$ and $^3/_{100}$.
  - The value of the third place to the right of 'one' is $^1/_{10}$ of $^1/_{10}$ of $^1/_{10}$ or $^1/_{1000}$ of 1. 0.001 is $^1/_{1000}$, 0.012 is $^{12}/_{1000}$ or $^1/_{100}$ and $^2/_{1000}$ and 0.345 is $^{345}/_{1000}$ or $^3/_{10}$, $^4/_{1000}$ and $^5/_{1000}$.
- A decimal point is used to separate the whole numbers on the left (the ones, tens, hundreds, etc.) from the decimal parts on the right (the tenths, hundredths, thousandths, etc.).

  Note: If using place-value charts, the decimal point does not hold a place

HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS
		•		

- Zero is essential as a place holder, for example 0.023 is not the same as 0.23. However, it is not written when the number makes sense without it, for example 0.230 is written as 0.23. The 0 is written, however, when it is necessary to show the accuracy of a measurement, for example, 2.230 metres indicates that the measurement has been made to the nearest millimetre.
- When selecting a context for developing an understanding of decimal place value, be aware of the difficulties that arise when using money or measurement. For example, $12.35 is usually treated as two whole number parts, 12 dollars and 35 cents rather than 12.35 dollars.

Resources

- Decimal number place value templates (see Appendix B), for each learner with four strips of equal length placed directly underneath each other: one left unmarked, one divided into tenths, one divided into hundredths and one divided into thousandths.

The guided teaching and learning sequence

1. Point to the strip divided into tenths and ask the learners what fraction one segment of the strip is. When the learners respond correctly by saying "one tenth", ask, "How do you write that?" Listen for "$^1/_{10}$", "tenth", "0.1" – ensure that all responses are included and written down.
2. Ask the learners to cover portions of the tenths strip, using the variety of possible names, for example, 0.2, seven tenths, $^4/_{10}$.
3. Ask the learners to answer questions such as:

   "Of 0.2 and 0.6, which is smaller and why?"
   "How else could $^{10}/_{10}$ be written?"
   "How else could $^{11}/_{10}$ be written?"
   "Put 0.6, 0.9, 1.1 in order and explain the reason for this order."

4. Point to the strip that is divided into hundredths and ask the learners what fraction one segment of the strip is. When the learners respond correctly by saying "one hundredth", ask, "How do you write that?" Listen for "$^{1}/_{100}$", "hundredth", "0.01" – ensure all responses are included and written down.
5. Ask the learners to cover $^{20}/_{100}$ and discuss the ways in which this could be written. Record all responses. Listen for and encourage "20 hundredths", "$^{20}/_{100}$", "0.20", "$^{2}/_{10}$", "0.2". Discuss the fact 0.20 and 0.2 are the same and that 0 is usually not written unless it is essential as a place holder or used to indicate a level of accuracy.
6. Ask the learners to answer questions such as :

   "Of 0.29 and 0.61, which is smaller and why"?
   "How else could 0.7 be written?"
   "Of 0.7 and 0.69, which is smaller and why?"
7. Point to the strip divided into thousandths and ask the learners what fraction is one segment of the strip. When the learners respond correctly by saying "one thousandth", ask, "How do you write that?" Listen for "$^{1}/_{1000}$", "thousandth", "0.001" – ensure that all responses are included and written down.
8. Ask the learners to cover $^{400}/_{1000}$ and discuss the ways in which this could be written. Record all responses. Listen for and encourage "400 thousandths", "40 hundredths", "4 tenths", "0.400", "0.40", "0.4", etc. Again discuss the role of 0 as a place holder.
9. Ask the learners to cover 0.3 on all strips and record all the ways it could be written. Ask them to explain their thinking (0.3, $^{3}/_{10}$, $^{30}/_{100}$, $^{300}/_{1000}$).
10. Repeat with 0.65, $^{250}/_{1000}$, 1.4.
11. Ask the learners to answer questions such as:

    "Of 0.294 and 0.615, which is smaller and why?"
    "Of 0.09 and 0.009, which is smaller and why?"
    "Of 0.8 and 0.699, which is smaller and why?"

Follow-up activity

Using the decimals sheet for support, ask the learners to put the following number in order from smallest to largest:

- 0.4, 0.05, 0.45, 0.448
- 3.387, 3.4, 3.09, 3.199.

Addition and subtraction facts

Number Facts progression, 2nd step

The purpose of the activity

In this activity, the learners develop strategies that will help them remember and recall the basic addition and subtraction facts. The learners focus on learning those facts they are not able to recall quickly.

The teaching points

- Basic facts for addition are the combinations where both addends are less than 10.
- Recalling basic facts is often referred to as 'mastery' and means that a learner can give a quick response (within about 3 seconds) without having to work out the fact by a method such as counting.
- Number relationships are the foundation for strategies that help learners remember basic facts.
- All of the addition basic facts are conceptually related, which means you can figure out new or unknown facts from those you already know. For example, if you know that 8 + 8 = 16, you can work out that 8 + 9 = 17 by adding one more.
- Subtraction facts correspond to addition facts. For example, 3 + 4 = 7, 4 + 3 = 7, 7 - 3 = 4, 7 - 4 = 3.
- Traditionally drill has been the most popular approach used in schools for students to learn to recall their basic facts. However, the very fact that some adult learners do not know their addition and/or multiplication basic facts suggests that drill alone does not work for many learners.
- It is important that the learners are not drilled in a basic fact until they at least have an efficient strategy for working it out. For example, if the learner has to count to work out 7 + 8 = 15, they are not ready to practise it for quick recall. Once they can work it out quickly, for example, by working from 7 + 7 = 14, then they could use drill or practice activities to develop mastery of the basic facts.
- Discuss with the learners the reasons why they need to be able to work out facts, rather than just rely on memory.

Resources

- Flash cards for the addition basic facts.
- Flash cards for the subtraction basic facts.
- Addition facts chart.

The guided teaching and learning sequence

1. First you need to find out the 'gaps' for each learner. Do this by 'testing' the addition basic facts, one at a time, using flash cards. If the learner responds quickly (within 3 seconds) and without obviously counting to solve the fact, place it in their 'known' pile. Continue with all the facts until they are sorted into two piles – those that are known and those that need to be learnt.

2. Give the learners an addition facts grid and show them how to record facts from their 'known' pile on the grid.

   For example: All the + 0 and + 1 and most of the + 2 facts and doubles.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4							
3	3	4	5	6						
4	4	5	6		8					
5	5	6	7			10				
6	6	7	8							
7	7	8								
8	8	9	10							
9	9	10								

Recording the known facts on the grid allows the learner to see the facts they know and the ones they need to learn. The focus should be on developing strategies to learn the unknown ones. The remainder of this sequence is presented as a series of ideas or approaches to help the learners fill the specific gaps in their quick recall of the addition basic facts. Rather than working through each idea, choose the ones that best suit the learner's gaps. As the learner builds on their mastery of addition facts, add these to the chart and to the 'known' pile of facts.

Facts with 0

Write out the 0 facts and ask the learner what they notice. They should observe that irrespective of whether the 0 is the first or the second addend, the result is the non-zero addend. For example 3 + 0 = 3.

One and two more than facts

The learners should be able to count on from the highest number to quickly find the answer to the facts.

Make a dice labelled (+1, + 2, −1, −2, +1, −1) and another dice labelled (4, 5, 6, 7, 8, 9). After each roll of the dice, the learner should immediately state the complete fact.

Double facts

The double facts seem to be relatively easy to learn and then form a good base for learning the near doubles. For example, 6 + 7 as double 6 and 1 more.

Make 5 and 10 facts

These facts are also relatively straightforward as they can be 'seen', using the fingers on one hand (5 facts) or two hands (10 facts).

Subtraction facts linked to addition facts

Subtraction facts tend to be more difficult to recall than the addition facts. Begin by sorting the subtraction facts into 'known' and 'unknown' piles for each learner. Ask the learners to take one of their 'unknown' facts and see if they can think which addition fact it is related to. For example, encourage them to see the link between 14 – 6 = 9 and 6 + 9 = 14 and therefore use known addition facts to recall subtraction facts.

Multiplication and division facts

Number Facts progression, 3rd step

The purpose of the activity

In this activity, the learners learn strategies that will help them remember and recall the basic multiplication and division facts. The learners focus on learning those facts they are not able to recall quickly.

The teaching points

- Basic facts for multiplication are the combinations where both factors are less than 10.
- Recalling basic facts is often referred to as 'mastery' and means that a learner can give a quick response (within about 3 seconds) without having to work out the fact by a method such as counting.
- Number relationships are the foundation for strategies that help learners remember basic facts.
- All of the multiplication basic facts are conceptually related, which means you can figure out new or unknown facts from those that are already known. For example, if you know that 5 x 6 = 30, you can work out that 6 x 6 = 36 by adding one more lot of 6 to get 36.
- Division facts correspond to multiplication facts. For example, 4 x 5 = 20, 5 x 4 = 20, 20 ÷ 5 = 4, 20 ÷ 4 = 5. The exception to this relates to the facts involving 0. While 0 ÷ 5 = 0, there is no real number or meaningful answer to 5 ÷ 0. As an example, if you have $20 to share equally between 5 people, then each person receives $4. You can use this to illustrate the problem of dividing by 0.

  Let's say you have $20 to share between 0 people and ask how much does each 'person' get? An attempt to calculate this is meaningless because the question itself is meaningless since there are simply no people to share anything with in the first place.
- Traditionally, drill has been the most popular approach used in schools for students to learn to recall their basic facts. However, the very fact that many learners are unable to recall their multiplication facts indicates that drill alone does not work for many people.
- It is important that the learners are not drilled in a basic fact until they at least have an efficient strategy for working it out. For example, if the learner has to skip-count or count in ones to work out 7 x 8 = 56, they are not ready to practise it for quick recall. Once they can work it out quickly, for example, by working from 6 x 8 = 48, then they could use drill or practice activities to develop mastery of the basic facts.
- There are hundreds of software programmes and websites that offer drill of basic facts. While these can help learners increase their speed of recall, you need to ensure that the learners first have efficient strategies for the facts included in the drill.
- Discuss with the learners the pros and cons of different ways of learning multiplication facts and any barriers they may have to learning them.

Resources

- A set of multiplication and division fact cards, preferably one set for each learner (masters available from the nzmaths website: www.nzmaths.co.nz/numeracy/materialmasters.aspx).
- Multiplication facts chart.

The guided teaching and learning sequence

1. First you need to develop an inventory of the known and unknown facts for each learner.

   a) This can be done by 'testing' each learner on the multiplication basic facts, one at a time, using flash cards. If the learner responds quickly (within 3 seconds) and without obviously skip-counting or counting in ones to solve the fact, place it in their 'known' pile. Continue with all the facts until they are sorted into two piles – those that are known and those that need to be learnt. (Note: This approach is also described in the "Deriving multiplication and division facts" activity on page 37.)

   b) Alternatively ask the learners to sort the facts into two piles and place in the 'unknown' pile those facts they are unsure about. Ask them to include any facts they solve by counting.

2. Give the learners a multiplication facts grid and show them how to record facts from their 'known' pile on the grid.

   For example:

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2		6		10				
3	0	3	6			15				
4	0	4				20				
5	0	5								
6	0	6								
7	0	7								
8	0	8								
9	0	9								

Recording the known facts on the grid allows the learner to see the facts they know and the ones they need to learn. The focus should be on developing strategies to learn the unknown facts. The remainder of this sequence is presented as a series of ideas or approaches to help the learners fill the specific gaps in their quick recall of the multiplication basic facts. Rather than working through each idea, choose the ones that best suit the learner's gaps. As the learner builds on their mastery of multiplication facts, add these to the chart and to the 'known' pile of facts.

Turn-around facts

It is very important the learners understand the commutative property, for example, that 4 x 3 gives the same result as 3 x 4. Although some facts seem easier to master than others, it is important that the 'turn-around' facts are learnt together as this halves the number of facts that need to be learnt.

Doubles

Facts that have 2 as a factor are equivalent to the addition doubles and should already be known by the learners. For example, if a learner knows 7 + 7 = 14 they should also know it as 2 x 7 = 14 and 7 x 2 = 14.

Zeros and ones

It is interesting to note that 36 of the 81 facts have either 0 or 1 as one of their factors. While these should be reasonably easy to learn, many learners seem to get confused by the 'rules' that are often associated with 0 and 1 facts. These facts are straightforward when learnt in a context. For example: if I have one bag of 6 apples, I have 6 apples in total.

Fives facts

Skip-counting in fives gives the learners practice in gaining a 'feel' for the numbers that are products of 5. The learners may also be able to connect to the fives facts if they think about them as being half of the tens facts.

continued...

x	0	1	2	3	4	5	6	7	8	9
0						0				
1						5				
2						10				
3						15				
4						20				
5	0	5	10	15	20	25	30	35	40	45
6						30				
7						35				
8						40				
9						45				

Nines facts

The nines facts can be some of the easiest facts to learn as there is an interesting and helpful pattern in the products.

- The digits in the product sum to 9.
- The digit in the tens place is one less than the number of nines you are finding. For example, 6 x 9 = 54 (where 5 is one less than 6).

Although the nines pattern helps some learners, others find it confusing. The nines can be also worked out by taking one group away from the tens facts. For example, 9 x 6 is 6 less than 10 x 6 = 60, giving 9 x 6 = 54.

x	0	1	2	3	4	5	6	7	8	9
0										0
1										9
2										18
3										27
4										36
5										45
6										54
7										63
8										72
9	0	9	18	27	36	45	54	63	72	81

The left-over facts

After learning the 0, 1, 2, 5 and 9 facts, there are just 25 left to learn, and 10 of these are turn-around facts. In the chart below, the 10 turn-around facts have been left unshaded. The 15 highlighted 'left-over' facts can be learnt by deriving them from a closely connected fact. For example 3 x 3 is 3 more than 2 x 3. This approach is explained in detail in the "Deriving multiplication and division facts" activity on page 37.

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3				9	12		18	21	24	
4				12	16		24	28	32	
5										
6				18	24		36	42	48	
7				21	28		42	49	56	
8				24	32		48	56	64	
9										

Division facts linked to multiplication facts

Division facts tend to be more difficult to recall than the multiplication facts. Encourage the learners to learn the related division facts with the multiplication facts.

Follow-up activity

There are hundreds of websites offering practice activities for basic facts. Here are two websites where the user can select the numbers and the operation(s) practised in the activity.

- ArithmAttack (www.dep.anl.gov/aattack.htm)
- The Skillswise "Times tables" games (www.bbc.co.uk/skillswise/numbers/wholenumbers/multiplication/timestables/game.shtml)

Estimating facts

Number Facts progression, 4th Step

The purpose of the activity

In this activity, the learners learn to apply their basic multiplication facts to problems involving multiples of tens, hundreds, thousands, etc. They learn that facility with these new number facts is central to making efficient estimations.

The teaching points

- There are three types of estimations in maths. (This activity focuses on computational estimation.)
  - Estimating measurements, for example, the length of a table.
  - Estimating quantities, for example, the number of people watching a basketball game.
  - Computational estimates, for example, estimating the answer to $24.95 x 18.
- A computational estimate involves finding an approximation to a calculation that you either cannot work out exactly or you do not need to work out exactly.
- An estimate is not a guess because an estimate involves using a strategy to find a good approximation to a calculation.
- Computational estimates involve using easier-to-handle numbers. For example, a computational estimate for 4,124 x 19 is 4,000 x 20. These easier-to-handle numbers are most often multiples of 10.
- Estimation strategies build on mental strategies where the largest value of a number is considered first.
- Discuss with the learners situations where computational estimations are used in everyday life (for example, comparing values in supermarket purchases, estimating the annual cost of power, estimating the cost of a proposed holiday, determining the reasonableness of a calculator computation).
- A consideration of the context of the problem is important when making sense of a computational estimate. For example, if the estimate involves the amount of concrete needed to cover a driveway, which of the following is reasonable: 0.5 m^3, 5 m^3, 50 m^3?

Resources

- Calculators.

The guided teaching and learning sequence

1. Write the following numbers on the board and ask the learners to find the one that is the best estimate of the number of days that a 10-year-old has lived.
   - 350 days
   - 3,500 days
   - 35,000 days
   - 350,000 days.
2. Discuss with the learners the reasoning they used to estimate 3,500 days.

   "Why did you select 3,500?" (350 x 10 = 3,500)
3. Check that the learners understand how to multiply a number by 10. Encourage them to explain the meaning as "multiplying by tens" rather than stating the rule "add a 0". While "add a 0" gives the correct answer, it doesn't convey a conceptual meaning unless it is linked to an explanation like "add a 0 in the ones place as there are no ones in the answer". For example:
   - 3 x 10 is 3 tens or 30
   - 25 x 10 is 25 tens or 250
   - 455 x 10 is 455 tens or 4,550.

continued...

4. If the learners seem unsure, pose several more problems where they multiple numbers by 10.
5. Ask: "What computation would you use to estimate how many days a 20-year-old has lived?"
6. Tell the learners to choose a computational estimate they could calculate quickly in their heads (without pen and paper or a calculator).
7. List the suggestions one at a time on the board. With each suggestion, ask the learner to state whether they think that the estimate would be 'over' or 'under' the answer.

300 x 20	under

8. After each suggestion, ask the learners to do the calculation in their heads. Ask for a volunteer to share how they worked out their answer.

   "How did you work out 300 x 20?" (I thought of it as 20 lots of 3 hundred, which is 60 hundreds or 6,000.)
9. Repeat with other calculation estimates, for example:
   - 350 x 20 = 7,000 (I thought of it as 700 x 10, which is 700 tens or 7,000)
   - 400 x 20 = 8000 (I thought of it as 20 lots of 4 hundreds, which is 80 hundreds or 8,000)
   - 3,500 x 2 = 7,000 (I thought of it as 2 lots of 3 thousand 5 hundreds, which is 7 thousands).
10. Discuss the estimates with the learners, encouraging them to see that the estimates will generally fall in a range around the exact answer.
11. Write 5,700 on the board. Have the learners work in pairs to write down computations that have 5,700 as the answer and that could be worked out mentally. Suggest they try to find a calculation for each of the number operations (multiplication, division, subtraction and addition).
12. Share ideas, asking the learners to explain how they worked out each problem mentally. For example:
    - 57 x 100 (57 hundreds)
    - 570 x 10 (570 tens)
    - 57,000 ÷ 10 (There are 5,700 tens in 57,000)
    - 5,000 + 700 (5 thousands and 7 hundreds)
    - 6,000 – 300 (60 hundreds take away 3 hundreds is 57 hundreds).

Follow-up activity

Ask the learners to work in pairs to find the missing factors or products in the following chart.

STARTING NUMBER	X	ANSWER
40	x 30	
600	x	1,800
30	x 70	
5,000	x 20	
90	x	630

When they have completed the given problems, ask the learners to take turns posing similar problems for a partner to solve.

Connecting percentages, decimals and fractions

Number Facts progression, 5th step;
Place Value progression, 5th step

The purpose of the activity

In this activity, the learners explore the connections between percentages, decimals and fractions and develop mental strategies for solving problems involving percentages.

The teaching points

- Percentages are another way of representing fractions and decimal fractions.
- Often the easiest way to find a percentage of a number mentally is to use the equivalent fraction.
- The learners also find out how to estimate percentages of numbers by choosing comparable fractions.

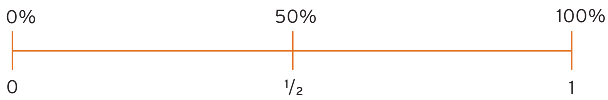
Resources

- Sheets of paper divided into 100 small squares (10 x 10).

The guided teaching and learning sequence

1. Ask the learners what 50% means. Possible responses include 50 percent, one-half, $^{50}/_{100}$, or 0.5. Ask the learners to explain how 50% becomes one-half. Listen for and reinforce that **percent** means "per hundred" and that 50 out of 100 ($^{50}/_{100}$) is equivalent to one half ($^{1}/_{2}$). Ask the learners to explain how 50% becomes 0.5. Listen for and reinforce that $^{50}/_{100}$ is 0.50, which is written as 0.5.

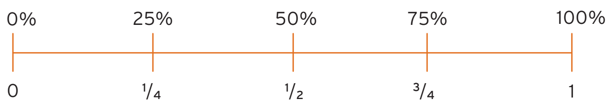
2. Give each learner a sheet of paper divided into 100 small squares (10 x 10) and ask them to fold this square in half and to check that the area of one half of the square is 50 out of 100. Show the learners how to record this on a number line and on a place-value table.



10s	1s	$^{1}/_{10}$ ths	$^{1}/_{100}$ ths	$^{1}/_{1000}$ ths	$^{1}/_{10,000}$ ths
	0 •	5	0		

3. Tell the learners they are going to fold the same square in half again. Ask them to predict how many parts this will make and what each part is called (one-quarter, $^{1}/_{4}$). Ask them to predict what one-quarter and three-quarters will be as percentages by predicting the number of squares out of 100 that will be in one- and three-quarters (25% and 75% respectively).

4. Ask the learners to fold the paper and check their predictions. Model or discuss how to add these to the number line and place-value table.



10s	1s	$^{1}/_{10}$ ths	$^{1}/_{100}$ ths	$^{1}/_{1000}$ ths	$^{1}/_{10,000}$ ths
	0 •	5	0		
	0 •	2	5		
	0 •	7	5		

continued...

5. Write the following fractions on the board, and ask the learners to express each of them as a percentage. Suggest learners use any representation (for example, 100-square paper, fraction/percentage number line or place-value table) they think will help them.

    - $^{1}/_{10}$
    - $^{3}/_{5}$
    - $^{3}/_{2}$ ($1\,^{1}/_{2}$)
    - $^{1}/_{8}$.

6. Ask the learners to discuss their solutions. Highlight the strategies they used, for example:

    - equivalent fractions: $^{1}/_{10} = {}^{10}/_{100} = 10\%$; $^{3}/_{5} = {}^{60}/_{100} = 60\%$
    - fraction/percentage number line: $^{3}/_{2} = 150\%$

    - halving

1	$^{1}/_{2}$	$^{1}/_{4}$	$^{1}/_{8}$	$^{1}/_{2}$
100%	50%	25%	12.5%	6.25%

7. At this point, consider asking the learners to explain how they can calculate GST by dividing by 8, or multiplying by 0.125 or halving, halving and halving again.

Follow-up activity

When the learners have completed the sequence described above, further consolidate their ability to convert between percentages and fractions by giving them the following percentages to convert to fractions: 60%; 90%; 37.5%; 175%; 250%.

Appendices

Contents

Appendix A

Appendix A.1 Budgeting in the supermarket

List of weights and prices for the brands on offer:

Bread brand A, 700g: $1.72

Bread brand B, 700g: $3.15

Bread brand C, 700g: $1.89

Milk brand A, 2L: $2.95

Milk brand B, 2L: $3.29

Milk brand C, 2L: $3.75

Butter brand A, 500g: $2.58

Butter brand B, 500g: $2.57

Butter brand C, 250g: $2.53

Chocolate chips brand A, 300g: $2.58

Chocolate chips brand B, 1kg: $7.90

Chocolate chips brand C, 250g: $2.53

PROBLEM	SOLUTION	STEP	PROGRESSION
1. Which milk is the cheapest?	Brand A		Number sequence: know the sequence of numbers, forwards and backwards, to at least 100.
2. How much cheaper is bread brand A than bread brand B?	$3.15 - $1.72 = $1.43		Additive: solve addition and subtraction problems involving decimals and integers, using partitioning strategies.
3. How many litres of milk can you buy if you have $10?	$10 ÷ $2.95 = 3 3 x 2L = 6L		Multiplicative: solve multiplication and division problems with single-digit multipliers or divisors mentally, using partitioning strategies and deriving from known multiplication facts.
4. How much will it cost to buy 1kg of butter brand A, two loaves of bread brand B, and 2L of milk brand A?	2 x $2.58 + 2 x $3.15 + $2.95 = $14.41		Additive: solve addition and subtraction problems involving decimals and integers, using partitioning strategies.
5. Which brand of chocolate chips is the best value?	Brand A pack = 0.86 per 100g = $8.60 per kg; brand B pack = $7.90 per kg; brand C = 4 x $2.53 = $10.12 per kg, so brand B is best value.		Multiplicative: solve multiplication or division problems with multi-digit whole numbers, using partitioning strategies.

Appendix A.2 Bone disease

Bone disease growing problem

WELLINGTON: Health researchers calling for taxpayer-funded bone density scans for older women who have small bone fractures want the Government to make osteoporosis one of its top priorities in public health.

New research released yesterday in Dubai shows osteoporosis is not only a "huge burden" on the health system — and is eroding the quality of life for many people — but also shows the nation's ageing population means this problem will worsen significantly by 2020.

The Burden of Osteoporosis in New Zealand (Bonz) report, released yesterday by International Osteoporosis Foundation chief executive Dan Navid, shows a hip fracture occurs somewhere in New Zealand every two hours.

The 80,000 people, 75% of them women, who will break bones this year because of osteoporosis, represent a fracture every six minutes. By 2020, with no change in Government policy and individual health patterns, this will be 115,000 people: a fracture every 4.5 minutes.

The Bonz report estimates the $300 million a year cost of treating these fractures is only part of the $1.15 billion health cost imposed by the condition, where bones become fragile and brittle, leading to a higher risk of fracture and breaks.

This year, osteoporosis will cost New Zealanders the equivalent of 12,000 years of life, due to disability imposed by the disease, but over half of it because of premature death.

Commissioned by Osteoporosis New Zealand, with money from a Fonterra education fund, the Bonz report was released with similar reports for Australia, Indonesia and the Philippines, with Mr Navid calling for bone health to be made a priority.

Osteoporosis NZ executive director Julia Gallagher said the report showed poor bone health is already a huge burden in New Zealand with devastating consequences for individuals and their families.

She also called on the Government for action on the "alarming spread and incidence of osteoporosis in New Zealand".

"Funded bone density scans should be available for all women [aged] over 50 years with a low trauma fracture."

The silent nature of the disease left many people unaware of the risk, with no symptoms until the first fracture.

State-funded bone scans of women suffering even a small fracture would give them the information needed to take preventive action and reduce their risk of future "osteoporotic" fractures.

A leading endocrinologist, Dr Brandon Orr-Walker, at Auckland's Middlemore Hospital, said the report's findings were a wake-up call for both the Government and doctors, but would also help to raise public awareness that poor bone health was not just an old person's issue.

"It is crucial for women and men to understand that after the age of about 25, when peak bone mass is reached, bones can be at risk of breaking down if they are not properly cared for," he said in a statement.

"To help keep bones strong, adults should give their bodies the right balance of weight-bearing exercise, nutrition and maintain a healthy lifestyle. Even if you grew up with a diet rich in dairy, it's vital you feed your bones the right nutrients at the right levels throughout adulthood to help maintain optimal bone strength and avoid the risk of debilitating problems later on in life."

Calcium is found naturally in foods including milk, yoghurt and cheese, as well as green leafy vegetables, some fish, citrus fruit, and beans.

People reach their peak bone mass between the ages of 20 and 30, and the higher the density at this time, the bigger the buffer they have later in life as bones lose their strength. Poor calcium intake or absorption, and a lack of vitamin D through limited exposure to sunlight, and smoking can all accelerate the bone loss.

The most common fractures involving osteoporosis occur in the hip, spine and forearm. Hips are the most costly, averaging $24,000 each for a hospital stay of 14 days in acute wards, with 70% of those patients requiring a further 22 days on a rehabilitation ward. About 50% of people suffering a broken hip require long-term care, and 25% die within a year. — NZPA

OSTEOPOROSIS

- A hip fracture occurs somewhere in New Zealand every two hours.
- The 80,000 people, 75% of them women, who will break bones this year because of osteoporosis, represent a fracture every six minutes.
- By 2020, with no change in Government policy and individual health patterns, this will be 115,000 people: a fracture every 4.5 minutes.
- Cost of treating these fractures estimated at $300 million a year.
- Most common fractures occur in the hip, spine and forearm.
- Hips are the most costly, averaging $24,000 each for a hospital stay of 14 days in acute wards, with 70% of those patients requiring a further 22 days on a rehabilitation ward.
- About 50% of people suffering a broken hip require long-term care, and 25% die within a year.
- To help keep bones strong, adults should give their bodies the right balance of weight-bearing exercise, nutrition and maintain a healthy lifestyle.

"Funded bone density scans should be available for all women [aged] over 50 years with a low trauma fracture"

	FOCUS QUESTION	ANSWER AND SOLUTION	STEP
1	About how many people break a hip each year?	12 (a day) x 365 = 4,380 people (365 x 10) + (2 x 365) = 4,380 Reasonable because 365 x 10 = 3,650 and 3,650 + 700 = 4,350	Multiplicative
2	About how many men break a hip each year?	25% of 4,380 = 1,095 4,380 ÷ 4: 4 x 1,000 = 4,000 4 x 90 = 360 4 x 5 = 20 Reasonable as $^1/_4$ of 4,000 is 1,000	Multiplicative
3	Do you agree that this represents a fracture every six minutes?	No, a fracture every 7 minutes is a better approximation. Every 6 minutes: That means 10 an hour or 240 a day 240 x 365 = 87,600 Every 7 minutes: 1,440 (mins per day) ÷ 7 = 205.7 a day 205.7 x 365= 75,080.5 75,000 is closer to 80,000 than 87,000	Multiplicative
4	Do you agree with the estimate of a fracture every 4.5 minutes by 2020?	Yes. 1,440 (mins per day) ÷ 4.5 = 320 a day. 320 x 365 = 116,800	Multiplicative

continued...

5	What was the average cost of treating a fracture?	$300 million ÷ 80,000 = 0.00375 million or $3,750.00) Reasonable as 80,000 x 4,000 = 320 million	Multiplicative
6	What proportion of the estimated $1.15 billion health cost of osteoporosis is spent on fractures?	0.3 ÷ 1.15 = 26% Reasonable as $^{0.3}/_{1.15}$ is similar to $^{3}/_{12}$ or $^{1}/_{4}$ (25%)	Multiplicative
7	What sense can you make of 12,000 years of life?	12,000 years x 365 = 4.38 million days 4.38 million days ÷ 80,000 people = 54.75 days per person or 12,000 years ÷ 80 years (approx life expectancy) = 150 lives	Multiplicative
8	How much per day is a hospital stay in an acute ward?	$24,000 ÷ 14 days = $1,714.29 per day Makes sense as 24,000 divided by 12 days is 2,000 a day	Multiplicative
9	About how many people who break a hip each year need long-term care?	50% of 4,380 = 2,190 4,380 ÷ 2 4,400 ÷ 2 = 2,200 20 ÷ 2 = 10 so 2,200 - 10 = 2,190 Reasonable as $^{1}/_{2}$ of 4,000 is 2,000	Multiplicative
10	About how many people who break a hip die within a year?	25% of 4,380 or $^{1}/_{2}$ of 2,190 (from Q9) = 1,095 4000 ÷ 4 = 1,000 400 ÷ 4 = 100 20 ÷ 4 = 5 so 1,100 - 5 = 1,095	Multiplicative
11	Does this mean that the risk of dying of a broken hip is 25%?	No. There are many other factors connected to the risk of dying. Most specifically the chance of breaking a hip increases with age. The chance of dying also increases with age!	Probability

Appendix A.3 Violence against police

More violence against police

AUCKLAND: Six police officers were assaulted every day last year as the number of attacks on the thin blue line rocketed to its highest level for a decade.

And one officer each day was seriously assaulted, according to the police annual report.

The report shows there were 2248 reported assaults on officers in the past financial year — including 88 using weapons.

That is a 6% increase on last year and up 17% from 1998, when 1924 assaults were recorded.

The number of serious assaults on police has risen by almost 20% over the past year — up from 331 to 393.

A decade ago, the figure for the same period was 234.

Police Association president Greg O'Connor said last night police were dealing with an ever more volatile world.

Attacked on duty

- Reported assaults on police last year: 2248
- Attacks involved weapons 88

National police spokesman Chester Borrows said the constant increases in police being assaulted was worrying — and underlined the need for Tasers to be introduced for frontline officers.

In total, 88 of the last year's assaults involved a weapon — 31 were firearms and 11 were with a stabbing or cutting weapon.

The increase in assaults on police echoes the rise in overall violent crime, which increased 4% in the past year.

Greg O'Connor

Mr O'Connor blamed the rises on substance abuse.

"A lot of it has got to do with P.

"More people are on it, so more people are prepared to have a go [at police].

"And there's much more booze around."

He also said there were more police on the streets and they had more interaction with the public, which "leads to a higher likelihood of assault and more opportunity for complaints".

Mr Borrows said the increasing use of weapons represented a growing threat to officers.

"I would expect the report from the Taser trial to recommend the use of them in New Zealand.

"They're a fantastic tool that prevents people getting shot and injured.

"It also prevents offenders being shot and injured — not just the police."

However, Green Party MP Keith Locke said Tasers would only be used in the assaults on police which involved weapons, and, despite the increase, the numbers did not justify it.

"There might be particular circumstances in which Tasers would be useful, but the downside in terms of undermining police relations with the community by the higher use of violence in policing is greater than those cases where a Taser would be justified."

A year-long pilot of the controversial Tasers ended in August.

A report on the pilot is expected in December, and a decision on their long-term use will be made in January.

There have been calls for Tasers to be introduced early.

New Zealand First MPs called for them to be expedited following the shooting by police of Stephen Bellingham last month, and a stand-off between police and a rottweiler dog which dodged numerous police bullets in an armed offenders' raid for firearms in Porirua.

The police annual report also showed the number of complaints against police rose 21% in the past year.

The annual report period does not cover the fatal shooting of Stephen Bellingham in Christchurch in September which the Police Complaints Authority is looking into.

There were 2768 complaints accepted for investigation in the past year.

However, the total of completed investigations upheld (8.7%) was less than the year before, when nearly 11% were upheld.

Conversely, the police survey of public trust and confidence in the police show the wounds from those incidents may be starting to heal — 71% of those surveyed said they had either full or quite a lot of trust and confidence in the police, up from 67% two years ago. — *The New Zealand Herald*

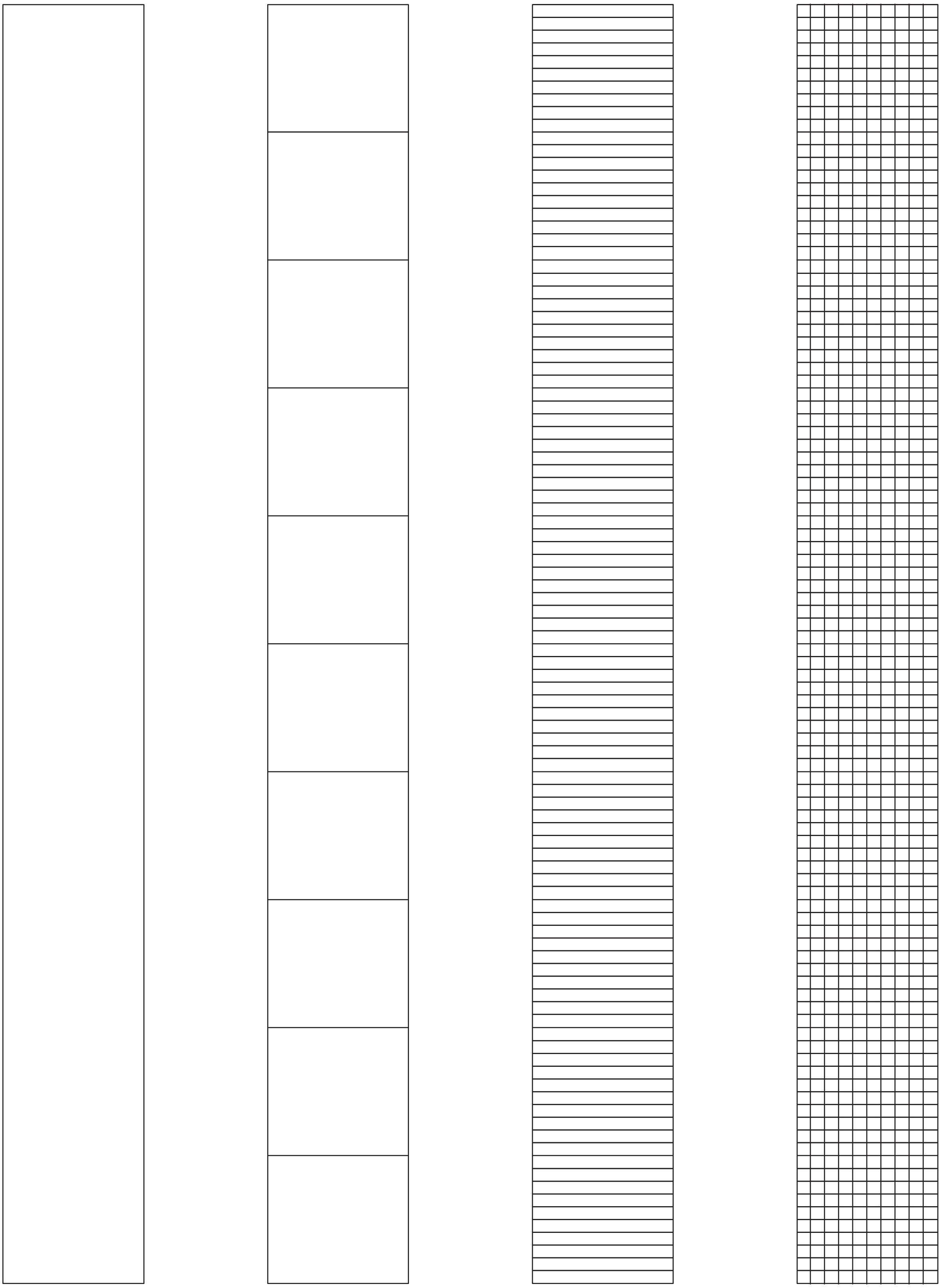
	FOCUS QUESTION	ANSWER AND SOLUTION	STEP
1	How many police officers were assaulted in 2006?	365 x 6 = 2,190 (300 x 6) + (60 x 6) + (5 x 6) = 2,190 Traditional algorithm or calculator with explanation: 2,190 makes sense as 6 x 400 is 2,400	Multiplicative
2	Explain why there appears to be a difference in the two paragraphs about the number of assaults.	The average of 6 is rounded from 6.1589041 (2,248 ÷ 365 = 6.1589)	Multiplicative
3	What percentage of assaults involved weapons?	88 ÷ 2,248 = 3.9% (calculator) This is reasonable as 10% of 2,248 is about 224 and 5% is about 110	Proportional
4	Explain how 2,248 is a 17% increase from 1924.	Calculate 17% of 1,924 and add this on to 1,924. Or Multiply 1,924 by 1.17	Multiplicative
5	How many police officers were assaulted in 2005?	2,248 ÷ 1.06 = 2,120.7 rounds to 2,121 assaults. This is reasonable as a 10% increase (212) from 2,121 is about 2,321 (adding 200)	Proportional
6	Do you agree that this is a 20% increase?	Yes. There were about 60 more serious assaults. $^{60}/_{330}$ is $^{6}/_{33}$, which is similar to $^{6}/_{30}$ or $^{1}/_{5}$ (20%)	Proportional

continued...

7	How many more serious assaults were there in 2006 compared to a decade ago?	393 - 234 = 159 159 is reasonable as 400 - 240 = 160	Additive
8	What percentage increase has there been in the last decade?	$^{159}/_{234}$ = 68% increase. 68% reasonable as $^{160}/_{240}$ = $^{2}/_{3}$ = 66%	Proportional
9	What percentage of assaults involved firearms?	31 ÷ 2,248 = 1.4% Reasonable as 1% of 2,248 is 22	Proportional
10	What percentage of assaults involving weapons involved firearms?	31 ÷ 88 = 35% Reasonable as $^{31}/_{88}$ is close to $^{30}/_{90}$ or $^{1}/_{3}$ (33.3%)	Proportional
11	How many complaints were made against police in 2005?	2768 ÷ 1.21 = 2,288 Reasonable as 10% of 2,288 is 228 so 20% is about 460. 2,300 + 460 is 2,760	Proportional
12	How many complaints were upheld in 2006?	8.7% of 2,768 = 241 Reasonable as 10% of 2,768 is 276	Proportional
13	How many complaints were upheld in 2005?	11% of 2,288 = 252 Reasonable as 10% of 2,288 = 229	Proportional

Appendix B

Appendix B: Decimal number place value template



Appendix C

Appendix C: Learner attitude questions

1. How much do you like maths?

HATE IT		OKAY		LOVE IT
1 ☐	2 ☐	3 ☐	4 ☐	5 ☐

2. How good do you think you are at maths?

AWFUL		OKAY		GREAT
1 ☐	2 ☐	3 ☐	4 ☐	5 ☐

3. How confident do you feel in doing maths during your daily life?

NOT AT ALL		OKAY		REALLY CONFIDENT
1 ☐	2 ☐	3 ☐	4 ☐	5 ☐

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